THE UNIVERSITY OF ALBERTA

EFFECT OF CHANGES IN RESERVOIR LEVEL ON THE STABILITY OF NATURAL SLOPES

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

EDMONTON, ALBERTA

SPRING, 1971

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled EFFECT OF CHANGES IN RESERVOIR LEVEL ON THE STABILITY OF NATURAL SLOPES submitted by Saeed Ullah Khan in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

This thesis deals primarily with the evaluation of the effects of the changes in reservoir water levels on the stability of natural slopes. For this purpose a detailed study of the steady state unconfined ground water flow through a porous medium is undertaken.

Finite difference techniques have been used to determine the phreatic surface and potentials in a slope consisting of a homogeneous, isotropic material with typical boundary conditions.

A Fortran IV program based on numerical techniques has been written to solve the steady state free surface flow with infiltration. The program is versatile and can be used for any geometric configuration and boundary conditions with or without infiltration. This program has been used in the evaluation of the amount of outflow from the banks into the reservoir at various reservoir water levels, in addition to determining the pore pressure data required for checking the stability of slopes.

Dimensionless curves have been developed for a typical geometric configuration of a slope which can be used for calculating the amount of outflow from the banks into the reservoir at various water levels, if the properties of the medium are known and if the initial head at the inside boundary is kept constant.

A hypothetical concept of constant discharge under

steady state seepage conditions which results in the raising of the initial head at the inside boundary with the raising of reservoir water level is postulated and graphs are developed to determine the change in head required to keep the ground water discharge constant at various reservoir water levels. The stability of slopes is then analyzed and results are compared with the cases of constant head.

The ICES computer program LEASE-1 based on the simplified Bishop slip circle analysis has been used for checking the stability of slopes.

The influence of infiltration on the phreatic surface and on the stability of slopes is examined. Typical results are compared.

Finally, on the basis of the constant discnarge concept, a possible explanation of Vajont rock slide is given.

ACKNOWLEDGEMENTS

The author wishes to thank Professor N.R. Morgenstern, who initially suggested the thesis topic, substantially helped and encouraged throughout the course of the study.

The author extends his sincere appreciation to Dr. Z. Eisenstein and Professor J.B. Nuttal for their critical review and helpful comments.

Thanks are also extended to Mr. Alok Jain of the Department of Chemical and Petroleum Engineering, who aided in the formulation of the computer program.

The financial assistance made available by the National Research Council is gratefully acknowledged.

The good efforts of Miss Susan Schultz in typing the thesis in its present form do not go unrecognized.

Last but not least, the helpful discussions with the colleagues in the Department of Civil Engineering are sincerely appreciated.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

1.1 Purpose of the Research

The stability of natural slopes subjected to seepage forces is dependent, among other factors, on the pore pressures within the soil or rock mass. The determination of these pore pressures in turn depends on the definition of flow domain.

A variety of slope stability problems associated with ground water flow demand a complete knowledge of the geohydrological characteristics of the area. The very occurrence and movement of the ground water which is an important phase of the hydrological cycle, depends mainly on the geological formation and the topography of the area. Almost all ground water is meteoric water derived from precipitation by the process of infiltration. The other main sources of ground water recharge are percolation from influent streams, lakes, reservoirs and glaciers. A schematic representation of the hydrological cycle, occurrence and movement of ground water is illustrated in Fig. 1.1.

In a natural landscape ground water moves under the action of gravity from higher to lower elevations. Without any external interference, a ground water basin is filled with water. The excess is discharged and it may emerge as surface water. Depending upon the geohydrological

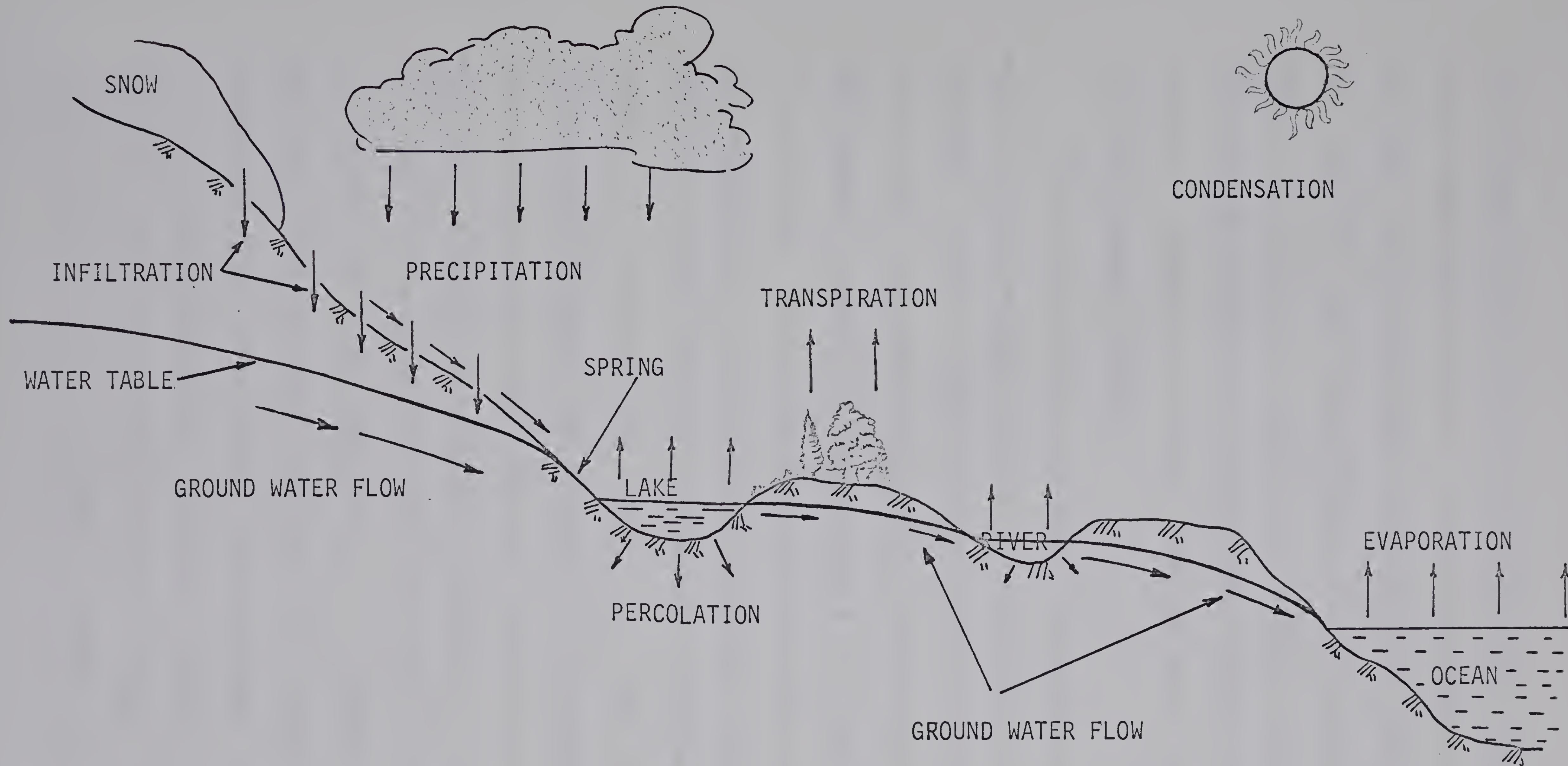


FIG. 1.1 THE HYDROLOGIC CYCLE (AFTER TODD, 1959)

conditions and the local topography the discharge of ground water tends to maintain a balance between the inflow and outflow. It is therefore reasonable to conclude that if the seasonal variations of the natural recharge are not very significant, an aquifer may yield a constant outflow. This may be evident as a head water source of a small stream or merely an effluent seepage which may evaporate from the ground surface.

The steady state unconfined flow pattern in a region may be determined by seeking the solution of Laplace's equation. This is done by using one of the available techniques with the prescribed initial and boundary conditions. The general practice of determining the free surface line is to fix the initial head well inside the slope and the exit conditions at the outside boundary. This initial head which is usually based on piezometric observations is chosen at a location well inside the slope and is assumed to be uninfluenced by any change in the geometric configuration of the slope and/or the exit conditions of the flow. However, in order to maintain a balance of inflow and outflow which is an inherent cycle of recharge and discharge of ground water, any change in exit conditions of flow at the outer boundary must influence the initial head at the inside boundary under steady state seepage conditions.

The impounding of a reservoir results in a change of the exit conditions of the natural ground water movement. If the initial head is assumed to remain constant for differ-

ent reservoir water levels, the phreatic surface would only shift with respect to this boundary condition.

This assumption of constant head leads to the conclusion that the raising of the reservoir water level would result in the reduction of the outflow, which is at variance with reality for most of the field cases under steady state seepage conditions.

Transient conditions are always on the safe side as far as the stability of slopes is concerned.

It therefore follows that the basic assumption of constant head does not hold for different reservoir water levels under steady state seepage conditions, whereas a concept of constant discharge seems to be more realistic.

In order to assess the qualitative and quantitative implications of this hypothesis on the stability of natural slopes, a complete study of the steady state free surface ground water flow with more realistic initial and boundary conditions, defining the actual flow domain in the field is necessary.

1.2 Review of Previous Work Done

The very occurrence and movement of ground water depends mainly on the geohydrological conditions of the area and the local topography. It is an important phase of the hydrological cycle and without any external interference maintains a balance between the inflow and outflow (Todd (1959); Linsley, J.R. *et al*. (1958)).

The problem of unconfined ground water flow is mostly treated as a two dimensional problem. Several techniques exist for solving the free surface problems of steady state unconfined saturated flow through porous media.

Earlier work is based on graphical methods. These methods of flow-net sketching known as Dupuit's solution (1863); Schaffernak and Van Iterson's solution (1917); L. Casagrande's solution (1932); and Pavlosky's solution (1931) are based on the Dupuit assumptions and give approximate solutions.

Some analytical solutions as reviewed by Harr (1962), they are provided by Polubarinova-Kochina (1962); Aravin and Numerov (1965); and Bear *et al*. (1968). All these methods become too complicated to be applied in field cases of difficult boundary conditions and geometric configurations.

Numerical techniques making use of high speed digital computers have been developed to overcome these difficulties. Finite difference methods using the relaxation techniques of Allen (1954); Shaw and Southwell (1941); McNown *et al*. (1953) and Thom and Apelt (1961) have been used by

various investigators in solving the steady state free surface flow problems.

Jeppson (1968); Finnemore and Perry (1968) have adopted these relaxation techniques in analyzing the seepage through an earth dam. Jeppson (1968) and Charmonman (1967) have also applied these finite difference methods in analyzing the seepage through canals or ditches.

The steady state free surface seepage problems have also been solved by Taylor and Brown (1967); Finn (1967); Zienkiewicz et al. (1966); Zienkiewicz, O. and Cheung, Y.K. (1965); Neuman and Witherspoon (1970) and Cheung et al. (1970) by the finite element method.

The numerical techniques given by the above mentioned authors using either finite difference or finite element methods may be used for defining the free surface flow without any infiltration over the free surface.

Polubarinova-Kochina (1962) has given an exact analytical solution for analyzing free surface seepage with constant infiltration over the entire free surface. Verruijt (1970) has also given an exact solution based on complex variable techniques. These solutions are again too difficult to apply to the field cases of complicated boundaries and geometric configurations.

The stability analyses of natural slopes subjected to seepage forces requires the knowledge of the geohydrology of the area, the properties of the material and the boundary conditions. The method of calculating the stability depends

on the properties of the material, the geometric configuration and the type of analysis, i.e., short term or long term stability analysis. It has been shown by Bishop (1952); Henkel and Skempton (1955) and Bishop and Bjerrum (1960) that the effective stress analysis is more advantageous when dealing with the long term stability of slopes.

A sufficiently accurate method based on effective stresses is given by Bishop (1955), who treats the failure surface as an arc of a circle. Although the Bishop simplified method does not satisfy fully statics, it has been shown that application of the Morgenstern and Price (1965) method to circular slip surfaces gives approximately the same results. If the material is homogeneous and isotropic, the Bishop simplified method may be used without any apprehension.

Almost all the studies dealing with the stability of a slope subjected to seepage forces adopt the general practice of fixing the initial head at the upstream boundary well inside the slope. This is generally based on piezometric observations and it is assumed that this head is not influenced by any change in the exit and/or geometric configuration of the slope.

As an example it is of interest to note the comprehensive work on the rock slide in the Vajont valley which has been reported by various investigators. All the detailed studies dealing with the stability of the Vajont rock slide after this unfortunate disaster are more or less based on

the assumption of constant head conditions.

Muller (1964, 1968) in describing the factors influencing the course of movement had considered the joint water pressure as a result of a sloping water table towards the lake. According to him, in 1961, the mountain ground water level sloped at an inclination of about 4:1 towards the side of the valley, corresponding to a hydrostatic thrust of the joint water on the creeping mass, estimated at about 2 to 4 million tons. This estimate is however based on the assumption that the slope of the ground water table remains the same even at higher lake levels. Later on, on the basis of limited piezometric observations it was concluded that complete levelling of the inclination of the ground water occurred.

Kenney (1967) in calculating the maximum unstabilizing effect of raising the reservoir level at Vajont, also assumed that the water level in the rock material prior to failure was horizontal and equal to the reservoir level.

Jaeger (1969) has discussed the stability of partly immersed fissured rock masses with reference to the Vajont rock slide. He has accepted the adverse influence of the sloping water table on the stability, but while analyzing the Vajont rock slide has assumed that the water level in the fissured rock is about the same as in the steady reservoir.

Since the overall geohydrology of the area was not known and the piezometric observations were very limited

(e.g., only three piezometric observation locations and the farthest one was located at an elevation less than 900 m. and approximately 800 m. away measured horizontally from the deepest point of the river (Muller (1964)), this assumption of levelling of water table was made without any due consideration of the influence of the filling of the reservoir on mountain ground water table further from the face of the slope.

During the first stage of filling, the sliding movements started at the reservoir elevation 635 m. and became quite high (8-10 cms/day) at the reservoir elevation 650 m. As pointed out by Skempton (1966), the reasons for this first sliding movements have to be explored. These movements stopped when the reservoir level was lowered to 600 m. and remained practically negligible during the period of constant reservoir level as well as during the second stage of filling up to the reservoir level of 650 m. The movements were magnified again when the previous reservoir level was crossed (see Muller, 1964, Fig. 19, p. 174). It is also evident from Muller, 1964, Fig. 19, p. 174 that no piezometric data of the area well inside the slope are available before October, 1961. Muller (1968) has explained this first time slide by considering an artesian thrust at the base of the sliding mass, which is based on the assumption that the strata lying under the slip surface were more permeable than the strata above.

Jaeger (1969) has explained this first time slide

by assuming low values of angle of shearing resistance along the upper strata and assuming the ground water table as horizontal.

Several authors have calculated the angle of shearing resistance at failure. Depending upon the profile chosen these values range from 17.5° to 24° (Muller (1968)). Nonveiller (1967) has obtained values of angle of shearing resistance at failure from 27° to 28°. As pointed out by Muller (1968) these values correspond to the position and shape of slip surface which differ very much from nature.

These values of angle of shearing resistance obtained at failure are much smaller than what could be expected for limestone. According to Skempton (1966) for limestone a friction angle of about 30° should be expected.

To summarize, it may be stated that all these excellent references have tried to explain this slide of very complex nature, but there are some questions which still remain unresolved.

1.3 Scope of the Study

This study is concerned with the determination of steady state unconfined ground water flow domains within a homogeneous, isotropic natural slope with representative boundary conditions. The boundary conditions are defined with special reference to the filling of reservoirs. This results in a change in flow domain and in turn, a change in pore pressures. The effect of this change in pore pressures at various reservoir water levels on the stability is evaluated.

The scope of the study also includes a summary of techniques available for solving free surface ground water flow problems.

Finite difference techniques for the numerical solution of Laplace's equation have been used to solve the free surface flow through porous media with constant infiltration.

The Fortran IV program given here is general and can also be used for determining free surface flow without infiltration. Typical results computed for the problem of seepage through a dam without infiltration are in close agreement with Numerov's analytical solution (Harr (1962)) (see Fig. 1.2).

The influence of raising of reservoir water level on the steady state seepage and consequently on the stability of slopes is studied in detail.

The constant discharge hypothesis which is more rea-

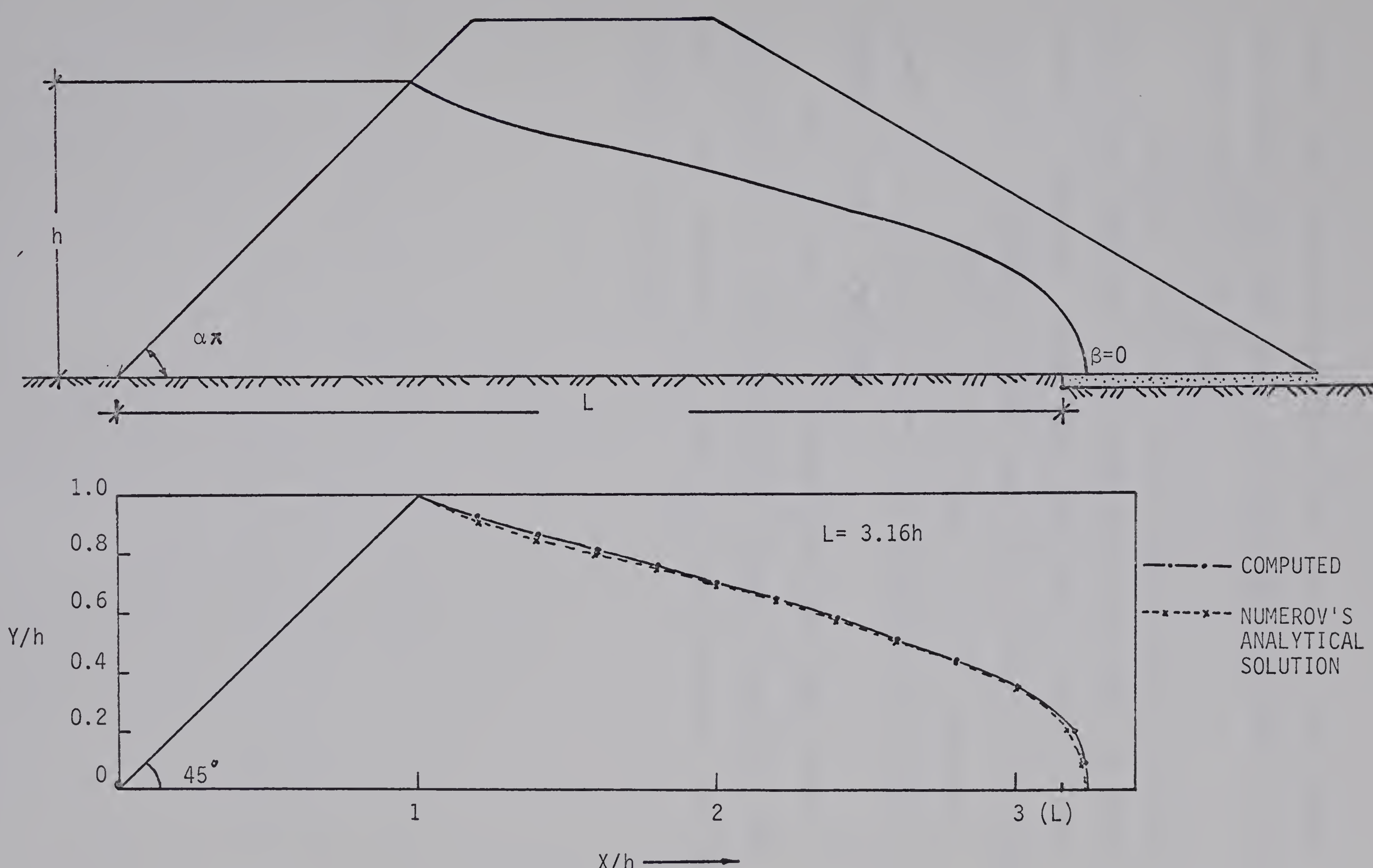


FIG. 1.2 COMPARISON OF ANALYTICAL AND NUMERICAL SOLUTIONS

listic than constant head for field cases, is postulated and studies have been made to assess the influence of this on the stability of slopes.

A study of outflow from the banks into the reservoir for various reservoir water levels is also made and dimensionless curves are presented, which can be used for calculating the seepage through dams with different tail water levels and/or outflow from the banks into the reservoir, knowing the properties of soil and keeping the initial boundary head as constant.

The influence of infiltration on the free surface flow and then on the stability of slopes is examined and results are compared with those of steady seepage conditions without infiltration.

CHAPTER II

HYDRAULIC CONCEPT OF FREE SURFACE GROUND WATER FLOW

2.1 General

The problem of unconfined ground water flow will be treated in two dimensions.

Assuming the seepage flow to be laminar, which is usually the case in practice, the applicability of Darcy's law is valid and the following fundamental equations for two dimensional flow in the xy plane are obtained.

$$u = \frac{\partial \phi}{\partial x} = - k \frac{\partial h}{\partial x} \tag{2.1}$$

$$v = \frac{\partial \phi}{\partial y} = - k \frac{\partial h}{\partial y} \tag{2.2}$$

where u and v are the components of discharge velocity in the x and y direction respectively.

Taking the y axis as vertical, oriented upward, and the x axis horizontal, the head h and velocity potential ϕ may be defined as

$$h = z + \frac{P}{\gamma_w} \tag{2.3}$$

$$\phi = -k(z + \frac{P}{\gamma_w}) + c \tag{2.4}$$

where z is the elevation head of the point above an arbitrary reference datum, P is the pressure in the fluid at that point, k is the coefficient of permeability and γ_w is the specific weight of water.

The continuity equation may be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.5}$$

Replacing u and v in the equation of continuity, the well known Laplace equation is obtained.

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{2.6}$$

Assuming the potential flow to obey Laplace's equation, the lines of equal velocity potentials are orthogonal to the flow lines. In ground water flow another harmonic function known as the stream function $\psi(x,y)$ is introduced, which is defined as

$$u = \frac{\partial\psi}{\partial y}$$

$$v = -\frac{\partial\psi}{\partial x} \tag{2.7}$$

The stream function $\psi(x,y)$ satisfies Laplace's equation if it is defined by the Cauchy-Riemann equations. The Cauchy-Riemann equations are obtained by combining equations (2.1), (2.2) and (2.7).

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$
$$\frac{\partial \phi}{\partial y} = - \frac{\partial \psi}{\partial x} \tag{2.8}$$

Since by definition it follows that the velocity potential is a single valued function at every point of the x-y plane, it is possible to draw lines of constant ϕ in the x-y plane. These lines are called potential lines and are generally drawn at constant intervals (Fig. 2.1).

$$\phi_1 - \phi_2 = \phi_2 - \phi_3 = \ldots \Delta\phi$$

Referring to Fig. 2.1 the directions s and n are defined by

$$s = x \cos \alpha + y \sin \alpha$$
$$n = - x \sin \alpha + y \cos \alpha \tag{2.9}$$

Simplifying and rearranging, we get

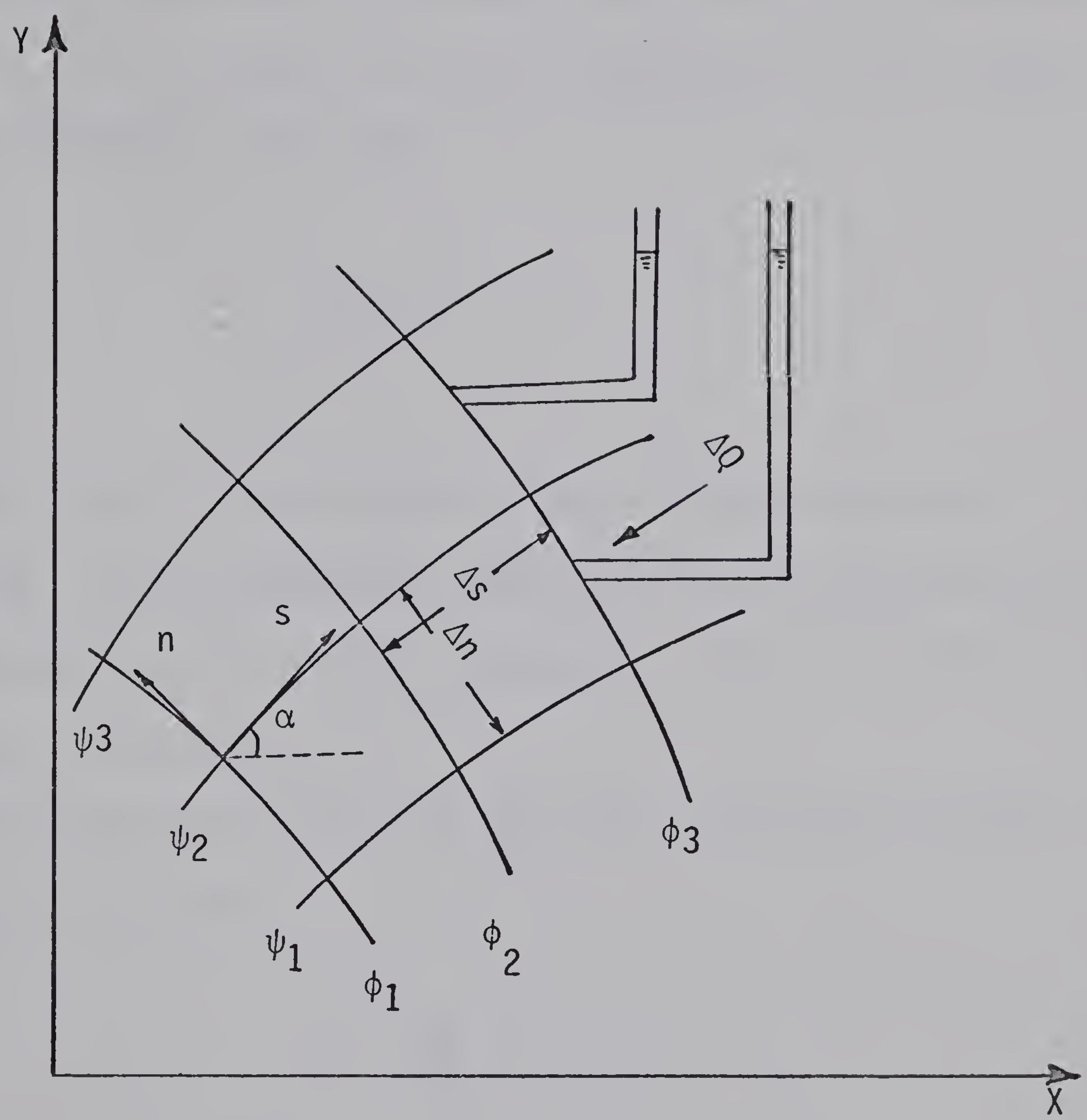


FIG. 2.1 REPRESENTATION OF POTENTIAL AND FLOW LINES

$$x = s \cos \alpha - n \sin \alpha$$
$$y = s \sin \alpha + n \cos \alpha \tag{2.10}$$

The angle α is chosen in such a way that the n-axis is tangent to the potential line and the s-axis is perpendicular to it. It follows from the tangency of the n-axis with the potential line that

$$\frac{\partial \phi}{\partial n} = 0 \tag{2.11}$$

Hence there is no specific discharge component in the n-direction. This obviously means that the direction of flow is perpendicular to the potential lines and flow occurs only in the s-direction.

If the magnitude of the specific discharge vector is denoted by q, we have

$$q = - \frac{\partial \phi}{\partial s} \tag{2.12}$$

The velocity components expressed in q are

$$q_x = q \cos \alpha$$
$$q_y = q \sin \alpha \tag{2.13}$$

Darcy's law may also be written as

$$q_x = -\frac{\partial\phi}{\partial x}$$
$$(2.14)$$
$$q_y = -\frac{\partial\phi}{\partial y}$$

and the continuity equation in terms of q becomes

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \qquad (2.15)$$

It follows from the continuity equation that the flow can also be derived from the single valued stream function $\psi(x,y)$ or simply ψ by

$$q_x = -\frac{\partial\psi}{\partial y}$$
$$(2.16)$$
$$q_y = +\frac{\partial\psi}{\partial x}$$

Since equation (2.15) is satisfied identically it follows from equation (2.14) that

$$\frac{\partial q_x}{\partial y} = \frac{\partial q_y}{\partial x} \qquad (2.17)$$

Substitution of equation (2.16) in (2.17) gives

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \tag{2.18}$$

which shows that ψ is also a harmonic function.

It is interesting to note that ψ does not vary in the s-direction, hence the direction of flow is tangent to the lines of constant ψ. This can easily be demonstrated as follows:

$$\begin{aligned} \frac{\partial \psi}{\partial s} &= \frac{\partial \psi}{\partial x}\frac{dx}{ds} + \frac{\partial \psi}{\partial y}\frac{dy}{ds} \\ &= q_y \cos\alpha - q_x \sin\alpha \\ &= q \sin\alpha \cos\alpha - q \cos\alpha \sin\alpha \\ &= 0 . \end{aligned} \tag{2.19}$$

Similarly

$$\begin{aligned} \frac{\partial \psi}{\partial n} &= \frac{\partial \psi}{\partial x}\frac{dx}{dn} + \frac{\partial \psi}{\partial y}\frac{dy}{dn} \\ &= - q_y \sin\alpha - q_x \cos\alpha \\ &= - q \sin^2\alpha - q \cos^2\alpha \\ &= - q. \end{aligned} \tag{2.20}$$

A comparison of equation (2.12) and (2.20) shows that

$$\frac{\partial \phi}{\partial s} = \frac{\partial \psi}{\partial n} = - q \ . \tag{2.21}$$

This means that in the x-y plane if lines of constant ϕ and ψ are drawn at intervals $\Delta\phi$ and $\Delta\psi$, these lines will form squares and if we choose

$$\Delta s = \Delta n$$

then

$$\Delta\phi = \Delta\psi \ .$$

The total discharge ΔQ between two stream lines at a mutual distance of Δn may be given by

$$\Delta Q = qB\Delta n = - B\Delta\psi \tag{2.22}$$

where B is the thickness of the plane where flow occurs.

This relationship shows that the discharge per unit thickness between two stream lines equals the difference in the values of ψ of these two stream lines and may be used in making outflow studies.

2.2 Mathematical Formulation of the Problem

The analysis of steady flow through porous media, involving the determination of the flow pattern and the potential distribution requires the solution of Laplace's equation subject to representative boundary conditions. This normally necessitates a detailed knowledge of ground water flow characteristics, the boundary conditions and the change in the flow pattern due to any changes in the initial or boundary conditions. It is therefore essential to define explicitly the initial and boundary conditions representative for the field case under study.

A hypothetical case of a natural slope consisting of homogeneous and isotropic material, subjected to various water pressures on the down stream side, corresponding to the filling of the reservoir is studied in detail (Fig. 2.2a). The following boundary assumptions are made:

(1) A known potential at the upstream boundary. This upstream boundary is generally selected at a location where the total head is assumed to be constant. This means that along the boundary AE the potential lines are vertical and the pressure distribution is hydrostatic.

(2) A known potential at the down stream boundary CD, which is also a line of constant potential.

(3) An impervious boundary ED where no flow takes place across it and thus ED is a locus of a stream line.

(4) Free water table AC, which is the upper limit of the seepage in the flow domain. Along this line the pres-

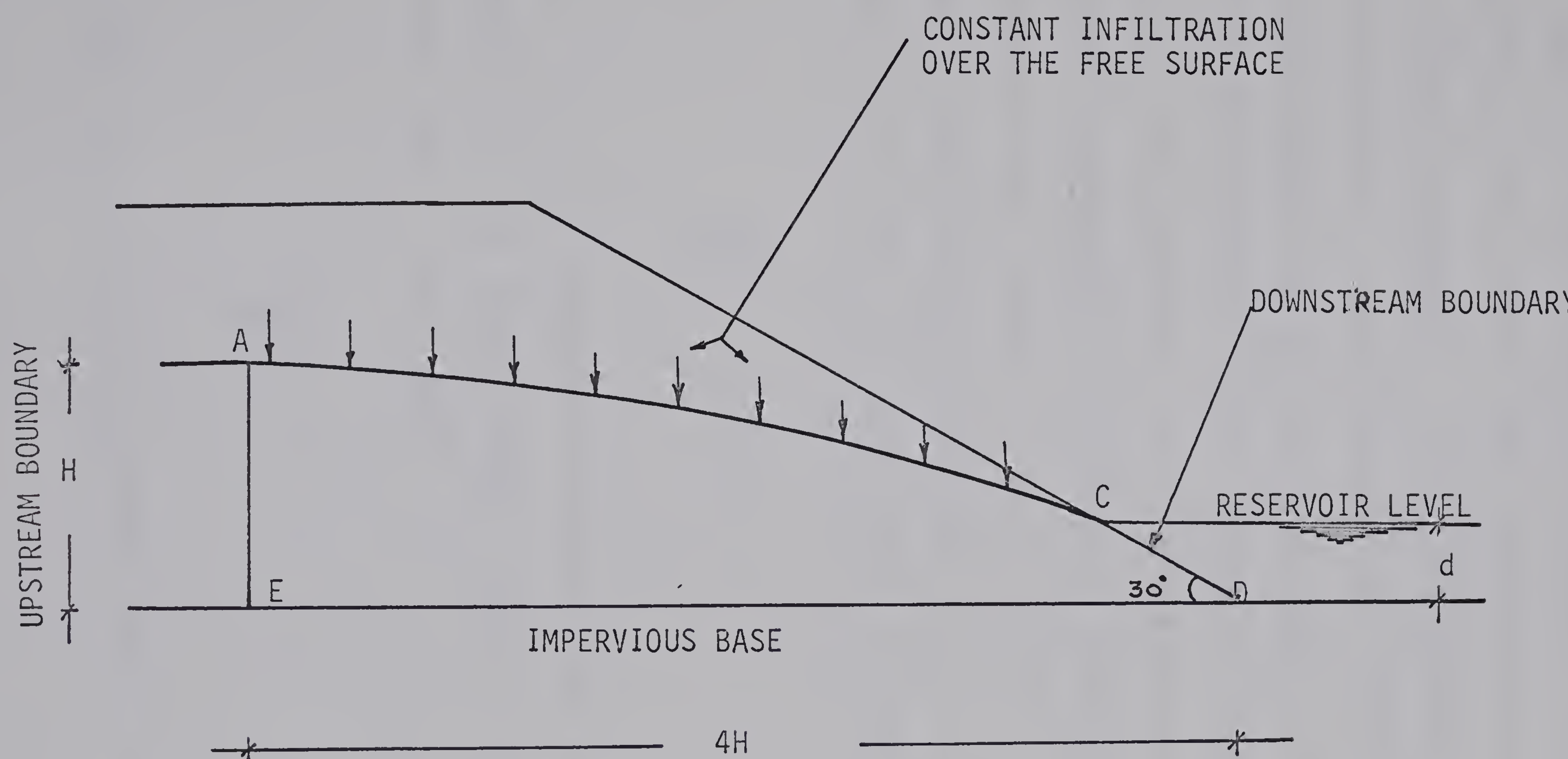


FIG. 2.2a BASIC LAYOUT OF SLOPE

sure at every point is equal to atmospheric pressure and that this free surface is a stream line.

(5) When dealing with a case of constant infiltration over the entire free surface some new boundary conditions have to be introduced. A detailed account of the typical boundary conditions and the derivation of the resulting governing differential equation for the free surface with infiltration is given as follows:

Assume a free surface line sloping at an angle α with the horizontal and having a constant infiltration over the entire surface (Fig. 2.2b(i) and Fig. 2.2b(ii)). If q is the volume of water reaching the free surface per unit area in the horizontal plane, per unit time, we have

$$\frac{\partial \phi}{\partial n} = - q \cos \alpha \qquad (2.23)$$

where α is the angle measured clockwise from the horizontal to the tangent of the free surface. When $\alpha = 0$, i.e., the free surface line is horizontal

$$\frac{\partial \phi}{\partial n} = - q \qquad (2.24)$$

when $\alpha = 90°$, i.e., the free surface line is vertical

$$\frac{\partial \phi}{\partial n} = 0 \qquad (2.25)$$

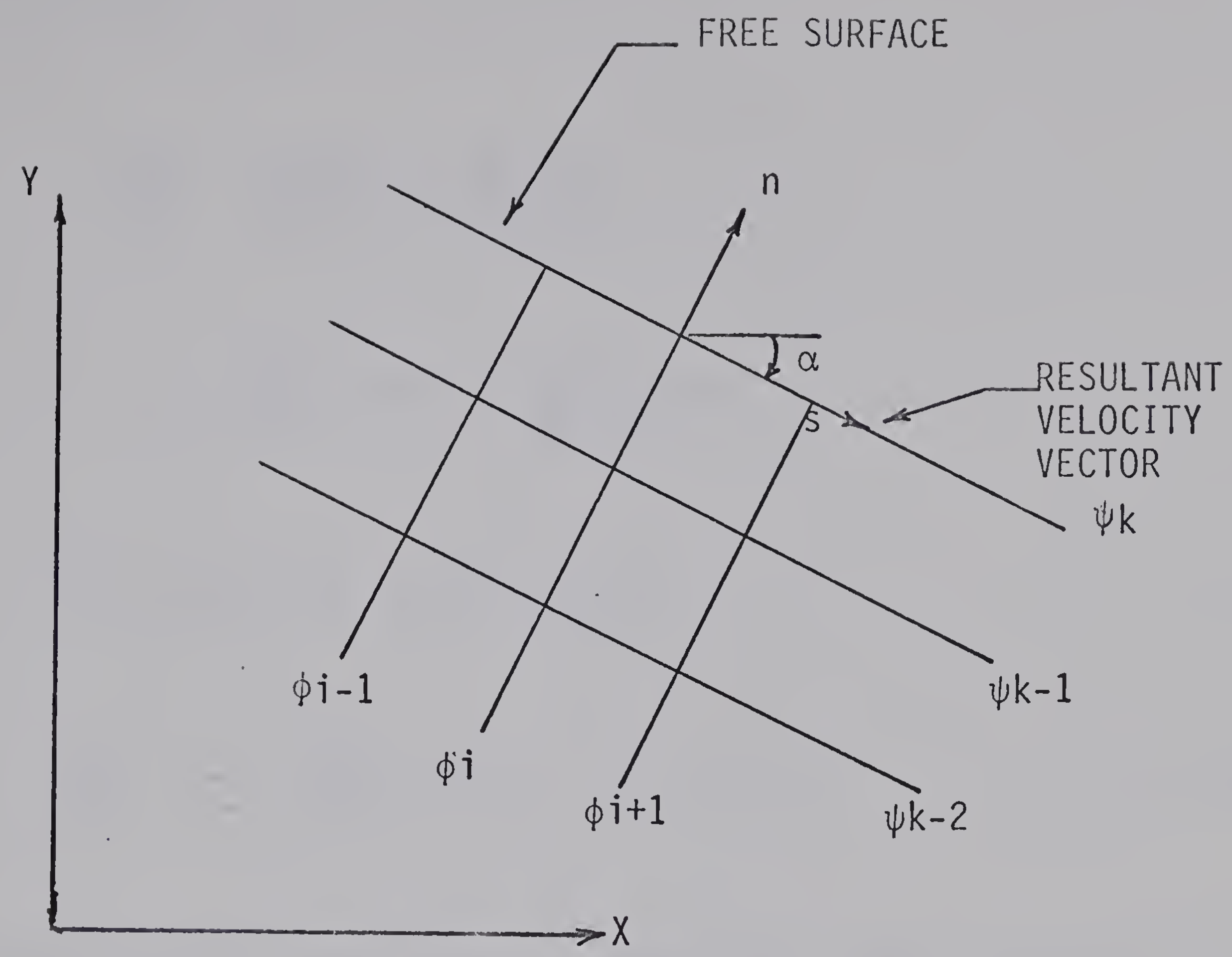


FIG. 2.2b(i) FREE SURFACE WITHOUT INFILTRATION

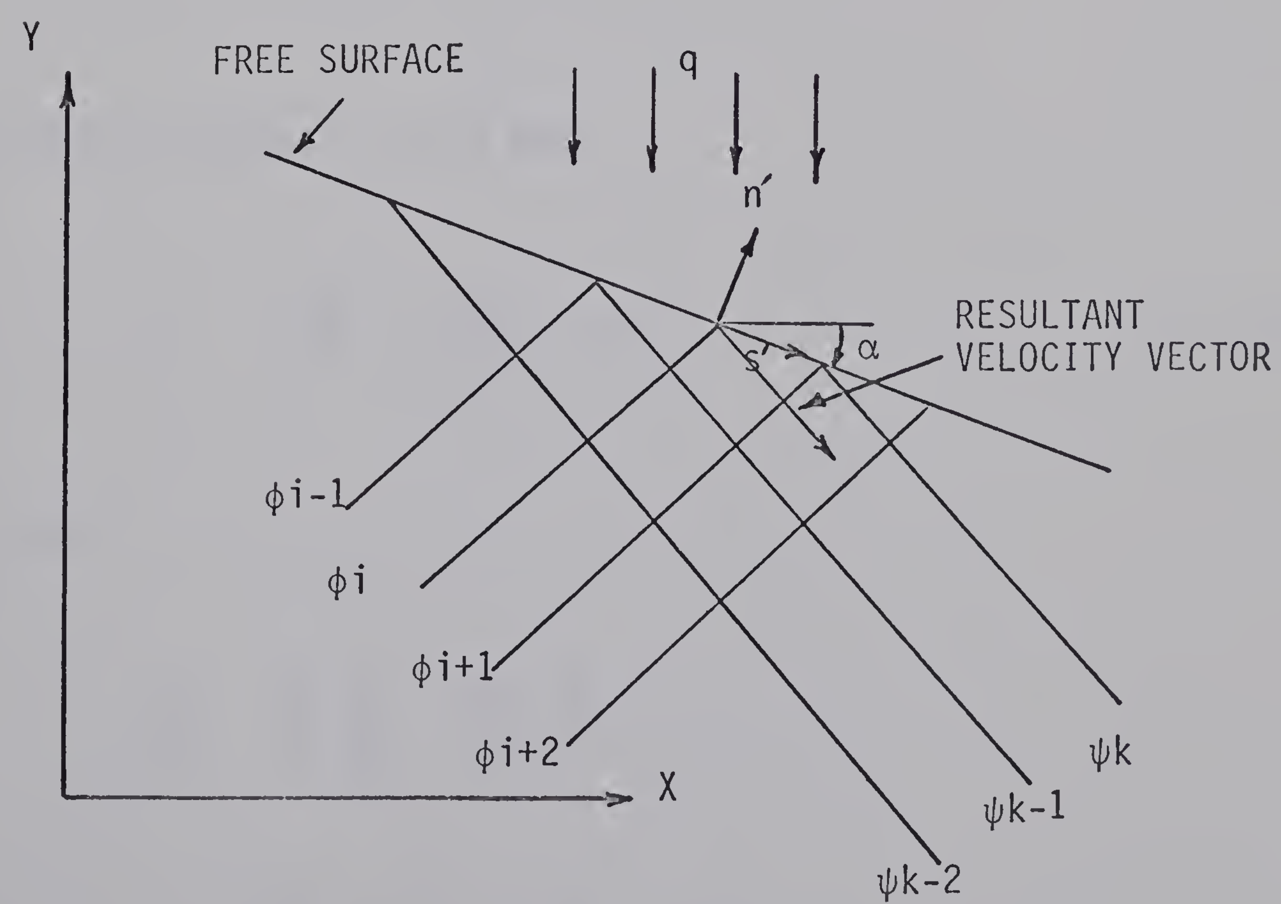


FIG. 2.2b(ii) FREE SURFACE WITH INFILTRATION

we also know that

$$\frac{\partial\phi}{\partial n} = \frac{\partial\phi}{\partial x}\frac{dx}{dn} + \frac{\partial\phi}{\partial y}\frac{dy}{dn}$$

$$= \frac{\partial\phi}{\partial x} \sin\alpha + \frac{\partial\phi}{\partial y} \cos\alpha$$

It follows from equation (2.23) that

$$\frac{\partial\phi}{\partial x} \sin\alpha + \frac{\partial\phi}{\partial y} \cos\alpha = - q \cos\alpha \tag{2.26}$$

we also know that at every point along the free surface line the pressure should be atmospheric, so that

$$\phi + ky = \text{constant} .$$

Differentiating along s, we have

$$\frac{\partial\phi}{\partial s} + k\frac{dy}{ds} = 0 \tag{2.27}$$

Again, we know

$$\frac{\partial\phi}{\partial s} = \frac{\partial\phi}{\partial x}\frac{dx}{ds} + \frac{\partial\phi}{\partial y}\frac{dy}{ds}$$

$$= \frac{\partial\phi}{\partial x} \cos\alpha - \frac{\partial\phi}{\partial y} \sin\alpha$$

Substituting in equation (2.27), we get

$$\frac{\partial \phi}{\partial x} \cos \alpha - \frac{\partial \phi}{\partial y} \sin \alpha = k \sin \alpha \tag{2.28}$$

Rearranging equations (2.26) and (2.28)

$$\frac{\partial \phi}{\partial x} \sin \alpha = - (\frac{\partial \phi}{\partial y} + q) \cos \alpha \tag{2.29}$$

and

$$\frac{\partial \phi}{\partial x} \cos \alpha = (\frac{\partial \phi}{\partial y} + k) \sin \alpha \tag{2.30}$$

It follows from equation (2.29) and (2.30)

$$\tan \alpha = - (\frac{\partial \phi}{\partial y} + q)/\frac{\partial \phi}{\partial x} \tag{2.31}$$

and also

$$\tan \alpha = \frac{\partial \phi}{\partial x}/(\frac{\partial \phi}{\partial y} + k)$$

Eliminating α from equation (2.29), we get

$$- (\frac{\partial \phi}{\partial x})^2 = k \frac{\partial \phi}{\partial y} + q \frac{\partial \phi}{\partial y} + (\frac{\partial \phi}{\partial y})^2 + qk$$

or

$$\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 + (k + q) \frac{\partial \phi}{\partial y} + qk = 0 \tag{2.32}$$

In the case of no infiltration equation (2.32) reduces to

$$\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 + k \frac{\partial \phi}{\partial y} = 0 \tag{2.33}$$

Equation (2.32) and (2.33), which apply along the free surface, are the governing differential equations for the free surface with or without infiltration respectively and are identical to the equations given by Polubarinova-Kochina (1962).

In terms of total head equation (2.26) may be written as

$$\frac{\partial H}{\partial x} \sin \alpha + \frac{\partial H}{\partial y} \cos \alpha = - Q \cos \alpha \tag{2.34}$$

where Q is defined as the ratio of the volume of water infiltrating per unit horizontal area per unit time divided by the coefficient of permeability of the soil, i.e., $Q = - q/k$.

2.3 Numerical Approach

Most of the free surface ground water problems with complicated geometry and/or boundary conditions become too cumbersome to be handled analytically. An alternative to analytic methods is the numerical approach, which reduces the problem to the solution of a set of algebraic equations.

It has already been demonstrated by Southwell (1946), Shaw (1953), McNown *et al*. (1953) and Allen (1954) that free surface problems can be handled by finite difference methods. Various schemes are available to transform the differential equations into a finite difference form. The Laplace equation is generally approximated by the following finite difference equation

$$H_c(I,J) = (H(I,J+1) + H(I-1,J) + H(I,J-1) + H(I+1,J)) * 0.25 \qquad (2.35)$$

where $H_c(I,J)$ represents the total head at the grid point (I,J) (Fig. 2.3). This finite difference equation has been obtained by using a five point grid system, a square network, and is therefore based on a second degree polynomial approximation. It is, however, possible to develop finite difference equations with smaller truncation errors Thom and Apelt (1961), but in view of the complicated algebra involved in combining the boundary conditions, the most promising approach seems to be a five point grid system.

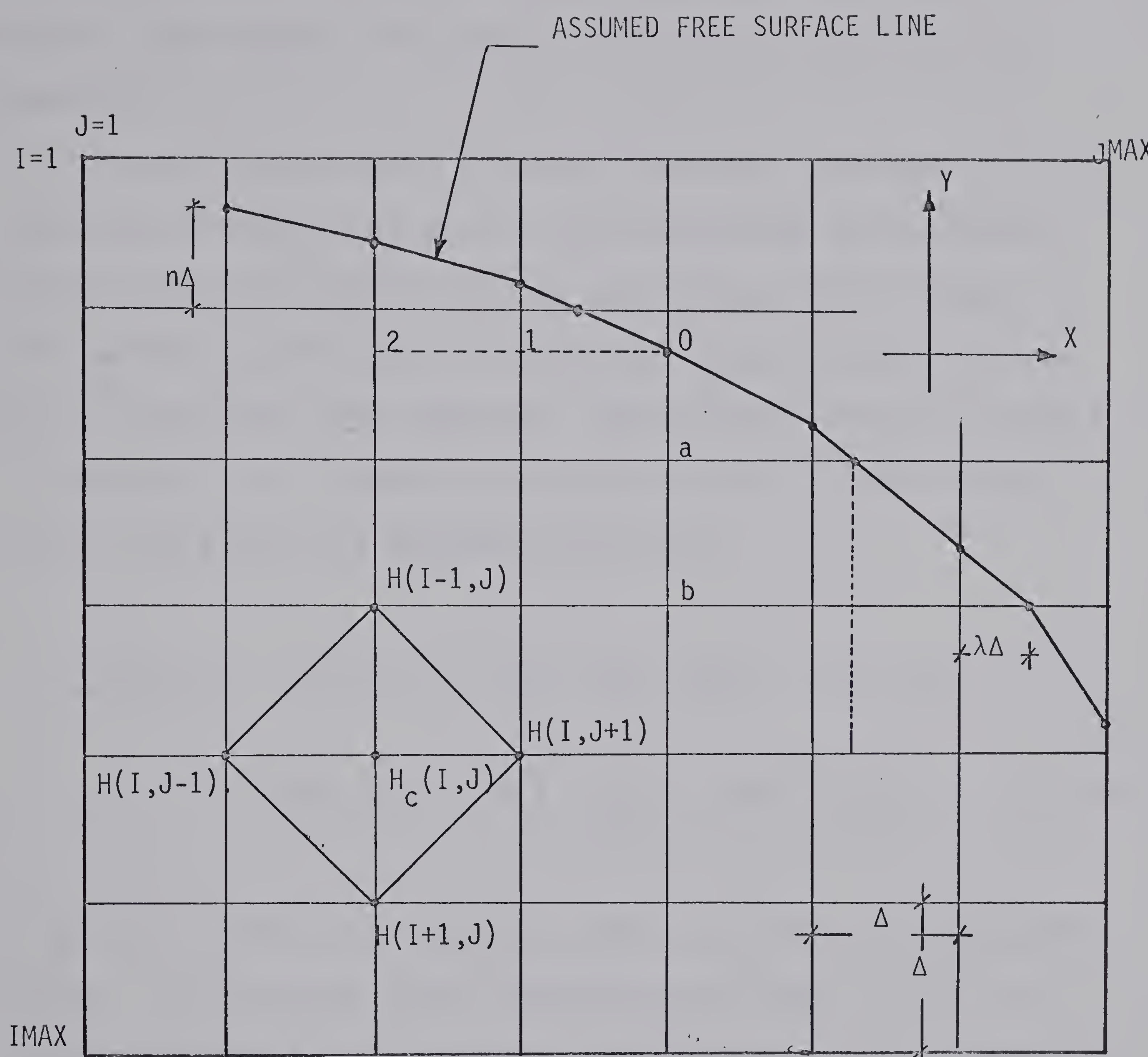


FIG. 2.3. REPRESENTATION OF COMPUTATIONAL MOLECULE AND FREE SURFACE LINE WITH IRREGULAR BOUNDARY

The finite difference equations can be solved by several methods including direct matrix inversion, the Gauss-Seidel type iterative procedure, etc. By these methods the lengthy "relaxation" solution is obtained repeatedly.

Since the solution of these equations requires an iterative process, the most efficient method is the Gauss-Seidel iterative method with an over-relaxation factor. This method ensures rapid convergence and thereby less number of iterations are required. This over-relaxation method is known as the "successive over-relaxation" (SOR) method and can be expressed mathematically as

$$H^k(I,J) = H^{k-1}(I,J) + \frac{\omega}{4} * [H(I,J+1) + H(I-1,J)$$

$$+ H(I,J-1) + H(I+1,H)] - \omega * H^{k-1}(I,J) \qquad (2.36)$$

in which k is the kth iteration and ω is the over-relaxation factor. It has been found theoretically that a value of ω ranging between 0 and 2 ensures convergence of the successive over-relaxation method. There is, however, an optimum value of ω which will give the most rapid convergence (Forsythe and Wasow (1960)).

The biggest advantage of the successive over-relaxation method is that it does not require any storage of the equations, but the value of the function is adjusted at every grid point by the finite difference equation. The new value

is used for all computations, involving the values obtained previously.

In order to determine potentials at the free surface, second degree polynomials were tried. These polynomials approximate the function between three consecutive points in each local interval. These points need not lie on the grid points shown on Fig. 2.3; in that case linear interpolation has been performed to obtain the values of potentials at these points.

The polynomial tried is of the form

$$H(x,y) = a + b_x(X - X_0) + C_x(X - X_0)^2$$
$$+ b_y(Y - Y_0) + C_y(Y - Y_0)^2 \qquad (2.37)$$

It follows from equation (2.37) that (with reference to Fig. 2.3)

$$H_0 = a$$

$$H_1 = H_0 + b_x(X_1 - X_0) + C_x(X_1 - X_0)^2 \qquad (2.38)$$

$$H_2 = H_0 + b_x(X_2 - X_0) + C_x(X_2 - X_0)^2 \qquad (2.39)$$

$$H_a = H_0 + b_y(Y_a - Y_0) + C_y(Y_a - Y_0)^2 \qquad (2.40)$$

$$H_b = H_0 + b_y(Y_b - Y_0) + C_y(Y_b - Y_0)^2 \qquad (2.41)$$

where X_1, X_2 and Y_a, Y_b correspond to the abscissa and ordinates of the points 1, 2 and a, b and X_0, Y_0 are the coordinates of the point 0 on the free surface (Fig. 2.3).

We now have four equations, i.e., (2.38), (2.39), (2.40) and (2.41) and five unknowns. Another equation is therefore required which is obtained from the boundary condition of the free surface.

Differentiating equation (2.37) with respect to x and y respectively, we get,

$$\frac{\partial H}{\partial x} = b_x + 2C_x(X - X_0) \tag{2.42}$$

and

$$\frac{\partial H}{\partial y} = b_y + 2C_y(Y - Y_0) \tag{2.43}$$

It follows from equations (2.42) and (2.43) that at point 0 on the free surface

$$\left.\frac{\partial H}{\partial x}\right)_0 = b_x \tag{2.44}$$

$$\left.\frac{\partial H}{\partial y}\right)_0 = b_y \tag{2.45}$$

Therefore equation (2.34) may be written as

$$b_x \sin \alpha + b_y \cos \alpha = - Q \cos \alpha \tag{2.46}$$

The angle α at each point may be defined as

$$\alpha = \tan^{-1} \left\{\frac{Y(J-1) - Y(J)}{X(J) - X(J-1)}\right\}$$

We now have five equations, e.g., (2.38), (2.39), (2.40), (2.41) and (2.46) which are set up in matrix form and solved for ϕ_0 by the Gaussian elimination method.

It may be seen that equation (2.34) can only be used for interior points when all the points lie on the grid point. However, the free surface points may not necessarily lie on the grid points (Fig. 2.3). In that case different finite difference equations based on a five point grid system with irregular boundaries are used. There may be three different cases corresponding to this irregular boundary (Fig. 2.3) which are described below:

(1) When the free surface point does not lie on the grid point but cuts the vertical grid line at a distance $n\Delta$ above the center grid point H(I,J) and all the other three points lie on the grid points, the following finite difference equation is used:

$$H_c(I,J) = \left[\frac{2H_S(J,1)}{n(n+1)} + \frac{2H(I+1,J)}{1+n} + H(I,J+1) + H(I,J-1)\right] * \frac{n}{2(n+1)} \tag{2.47}$$

in which HS(J,1) represents the potential at the respective points on the free surface (Allen (1954)).

(2) When the free surface point cuts the horizontal grid line at a distance $\lambda\Delta$ from the center grid point H(I,J) and the other three grid points are at a distance Δ each from the point H(I,J), the finite difference equation used is of the following form

$$H_c(I,J) = \left[\frac{2H_S(J,1)}{\lambda(1+\lambda)} + \frac{2H(I,J-1)}{1+\lambda} + H(I-1,J) + H(I+1,J)\right] * \frac{\lambda}{2(\lambda+1)} \tag{2.48}$$

(3) Lastly when the free surface line cuts the vertical as well as horizontal grid lines at a distance $n\Delta$ and $\lambda\Delta$ respectively from the center grid point, the following finite difference equation is used

$$H_c(I,J) = \left[\frac{2H_S(J+1,1)}{\lambda(1+\lambda)} + \frac{2H_S(J,1)}{n(1+n)} + \frac{2H(I,J-1)}{(1+\lambda)} + \frac{2H(I+1,J)}{1+n}\right] * \frac{n\lambda}{2(n+\lambda)} \tag{2.49}$$

The problem of the impervious boundary is treated on the basis of the fact that there is no flow across the boundary. The finite difference equation obtained for this case is of the following form:

$$H_c(I_{MAX},J) = [H(I_{MAX},J+1) + 2H(I_{MAX-1},J) + H(I_{MAX},J-1)] * 0.25 \tag{2.50}$$

2.4 Method of Solution

The correct position of the free surface is obtained systematically by seeking the solution of the finite difference equations corresponding to the various boundary conditions.

A Fortran IV program has been written to solve this steady state free surface ground water flow problem. This program determines the correct position of the free surface in addition to the potentials at the grid points.

A systematic scheme has been postulated which is outlined stepwise as follows.

Step 1

The location of the free surface is estimated and its coordinates are defined on the grid lines. Initial values of potentials at the free surface, at the interior grid points and also at the prescribed upstream and downstream boundaries are given.

Step 2

Laplace's equation, approximated by finite difference equations is solved by successive over-relaxation subject to the boundary conditions. The residue at every center grid point of the five point grid system is determined and checked with the maximum allowable value. The iterative process is continued until this condition is satisfied.

Step 3

The values of potentials for the two consecutive points near the free surface which are not lying on the grid points in either the X or Y directions are interpolated and are used in solving equations (2.38), (2.39), (2.40), (2.41) and (2.46).

The five simultaneous linear equations are solved by Gaussian elimination method and the potentials at points on the free surface are obtained.

Step 4

Using these values of potential at the free surface, the potentials at the interior points are then corrected by using SOR.

The process of relaxation is continued till the specified value of the residue is achieved. Again the potentials at free surface are determined in a similar way as described above.

These potentials at the surface are compared every time with those obtained from the previous iteration and the whole operation is repeated until the difference in potentials at all the free surface points is equal to or less than the specified value.

Step 5

The last step is to check the boundary condition at the free surface. The pressure at the free surface should

be atmospheric, or taken to be equal to zero.

We know

$$H = \left(\frac{P}{\gamma_w} + y\right)$$

or

$$P = \gamma_w(H - Y)$$

where H = total head or piezometric head

Y = elevation head.

For the condition $P = 0$, H must be equal to Y. This condition is checked at every point on the free surface and if it is not satisfied, the value of Y is changed at the respective points. This means a new trial free surface is drawn and the potentials at the free surface are determined. If the values of these potentials thus obtained at the respective points on the free surface are equal to the values of Y initially given, the problem is solved and this line represents the correct free surface.

This method which appears to be lengthy is quite rapid. The trial free surface lines converge very rapidly and the final solution is obtained in three or four trials.

CHAPTER III

OUTFLOW STUDY AND STABILITY ANALYSES

3.1 General

The amount of outflow from a natural slope is dependent on, among other things, the permeability, the slope of the ground water table and the boundary conditions. The most common practice is to fix the upstream head and boundary on the basis of some piezometric observations prior to the filling of the reservoir. This upstream boundary condition is generally kept unchanged in determining the flow pattern corresponding to various reservoir water levels. If the slope of the initial equilibrium water table is not very steep and the upstream boundary is chosen at a location well inside the slope, it might be a fair approximation to assume that the head H at the upstream is not influenced by the changes in the reservoir water levels. On the other hand if the geometric configuration of the slope and the geohydrological conditions of the area are such that the initial slope of the natural ground water table is quite steep, the assumption of constant head for determining the pore pressures at various reservoir water levels does not seem to be realistic.

Various methods are available for analyzing the stability of slopes. Studies have shown that effective stress analyses are more useful in analyzing the long term stability

of slopes and embankments (Bishop, 1952; Henkel and Skempton, 1955; Bishop and Bjerrum, 1960). It therefore follows that in order to use this analysis, it is necessary to know the pore pressure distribution within the soil or rock mass.

A sufficiently accurate method of stability analysis based on effective stresses is given by Bishop (1955) and is known as Bishop's simplified method. This method treats the failure surfaces as arcs of circles. It has been adopted in this study.

3.2 Outflow Study

The main aim of this part of the study is to evaluate the reduction in discharge from the bank with the raising of reservoir water level if the head at the upstream boundary is kept constant. The case of impounding behind a dam, where the reservoir water level is kept constant is analogous to it. Since the influence of the upstream slope of a dam is small on the seepage discharge (Harr (1962)), for practical purposes, this study may also be found useful in calculating seepage through a dam at various tail water levels.

For this purpose a hypothetical case of a 30° slope with an impervious base, consisting of a homogeneous and isotropic material has been adopted. The upstream head is fixed at an elevation of 1000 and the reservoir water level is raised from 0 to 800. The upstream boundary is chosen at a distance four times the head inside the bank and is assumed to be uninfluenced by the changes in the reservoir water levels. The computer program has been used to determine the free surface and potentials for each case of reservoir water level, ranging from 0 to 800 elevation in increments of 200 (Fig. 3.1 through 3.5). Having obtained the free surface line and the potentials, the equipotential and flow lines are drawn and the amount of outflow in a dimensionless form is calculated for each case. A sample calculation of q_{max} for the case d/H = 0 is given in Appendix B.

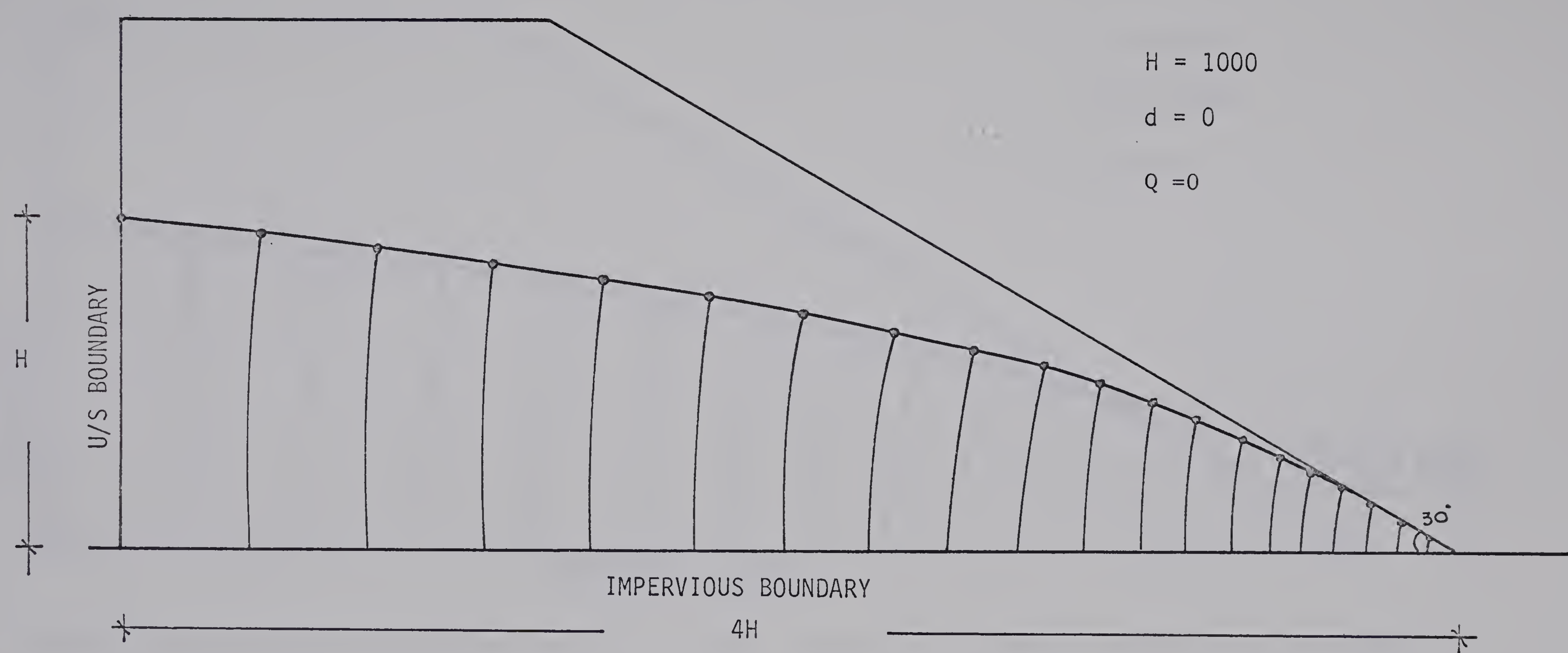


FIG. 3.1   FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

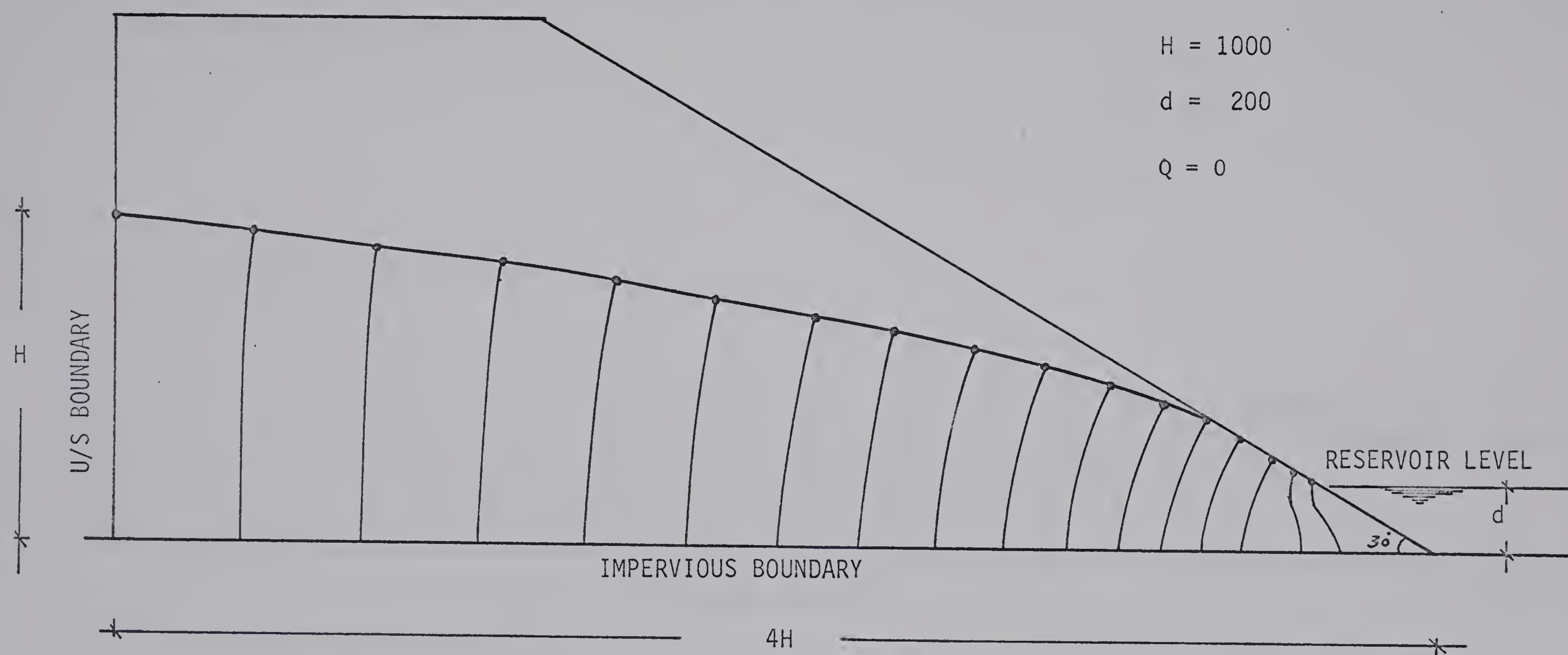


FIG. 3.2 FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

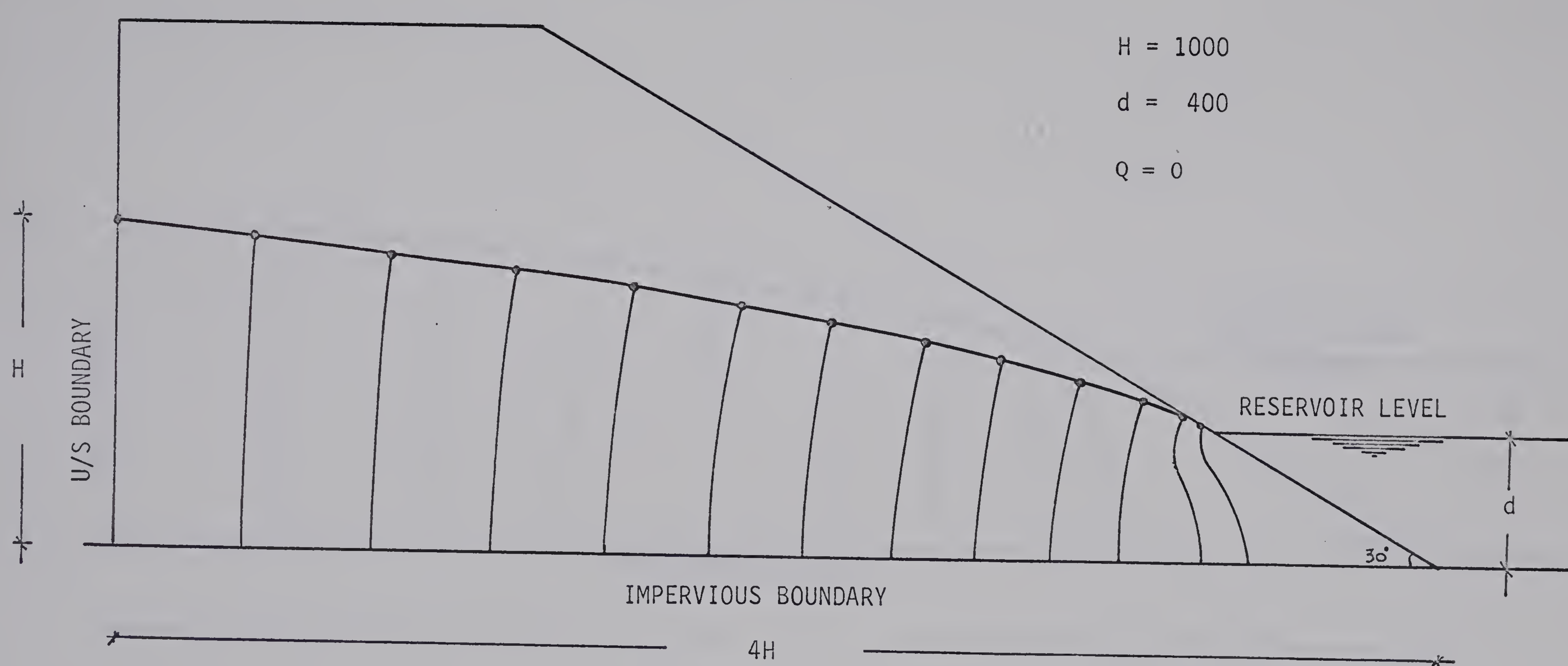


FIG. 3.3 FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

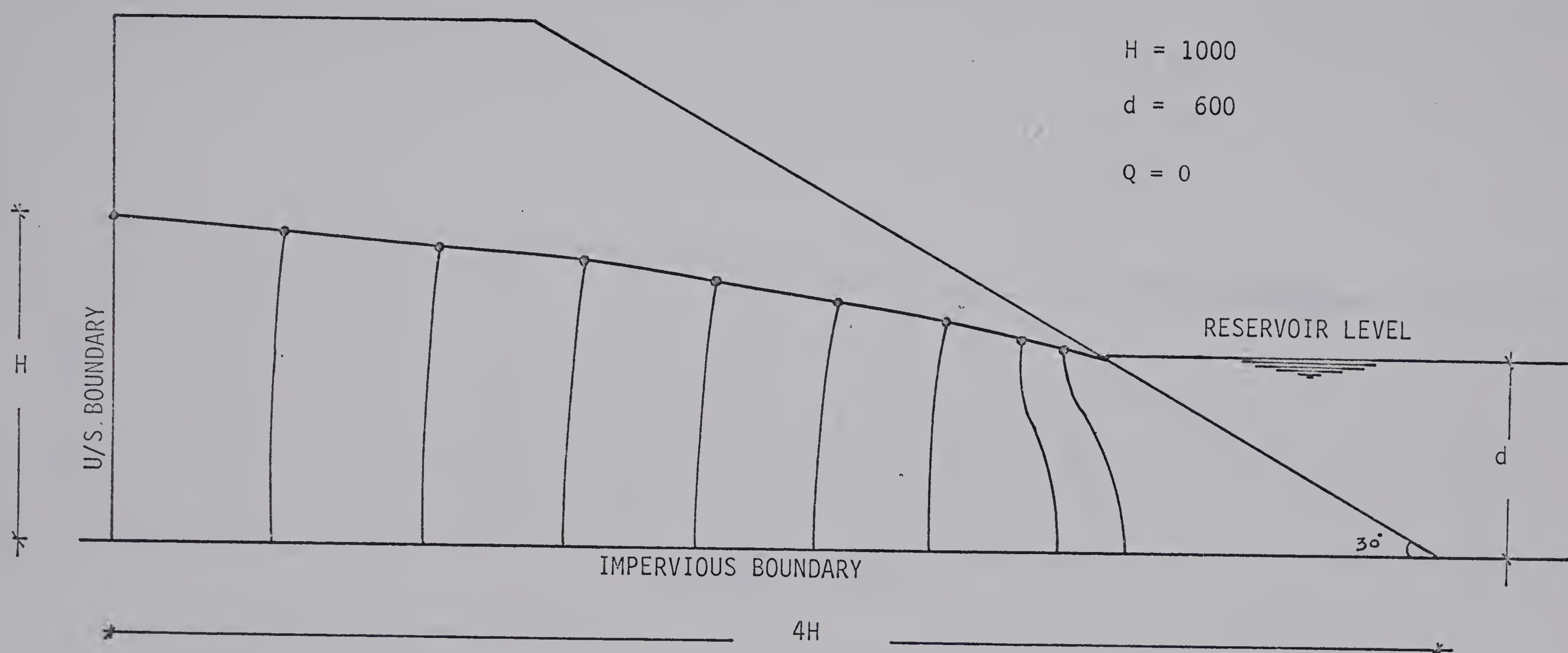


FIG. 3.4 FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

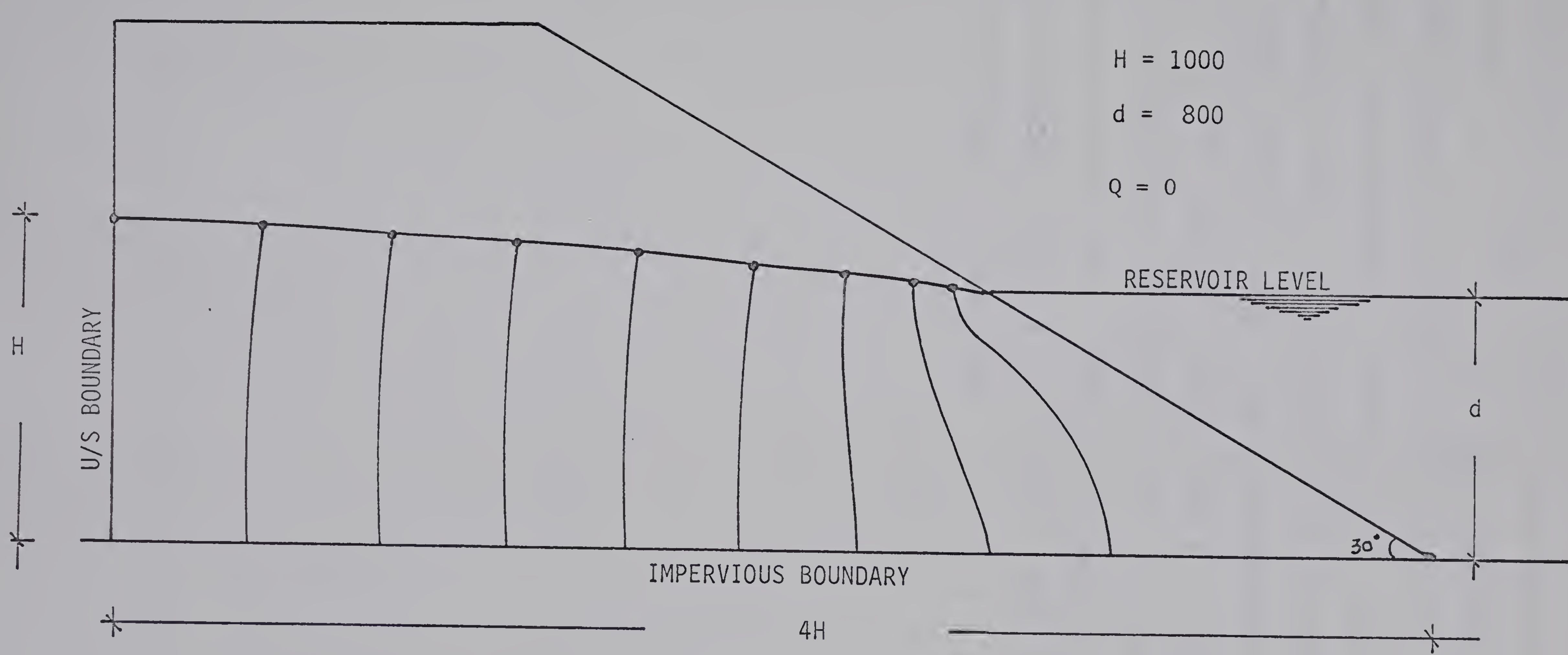
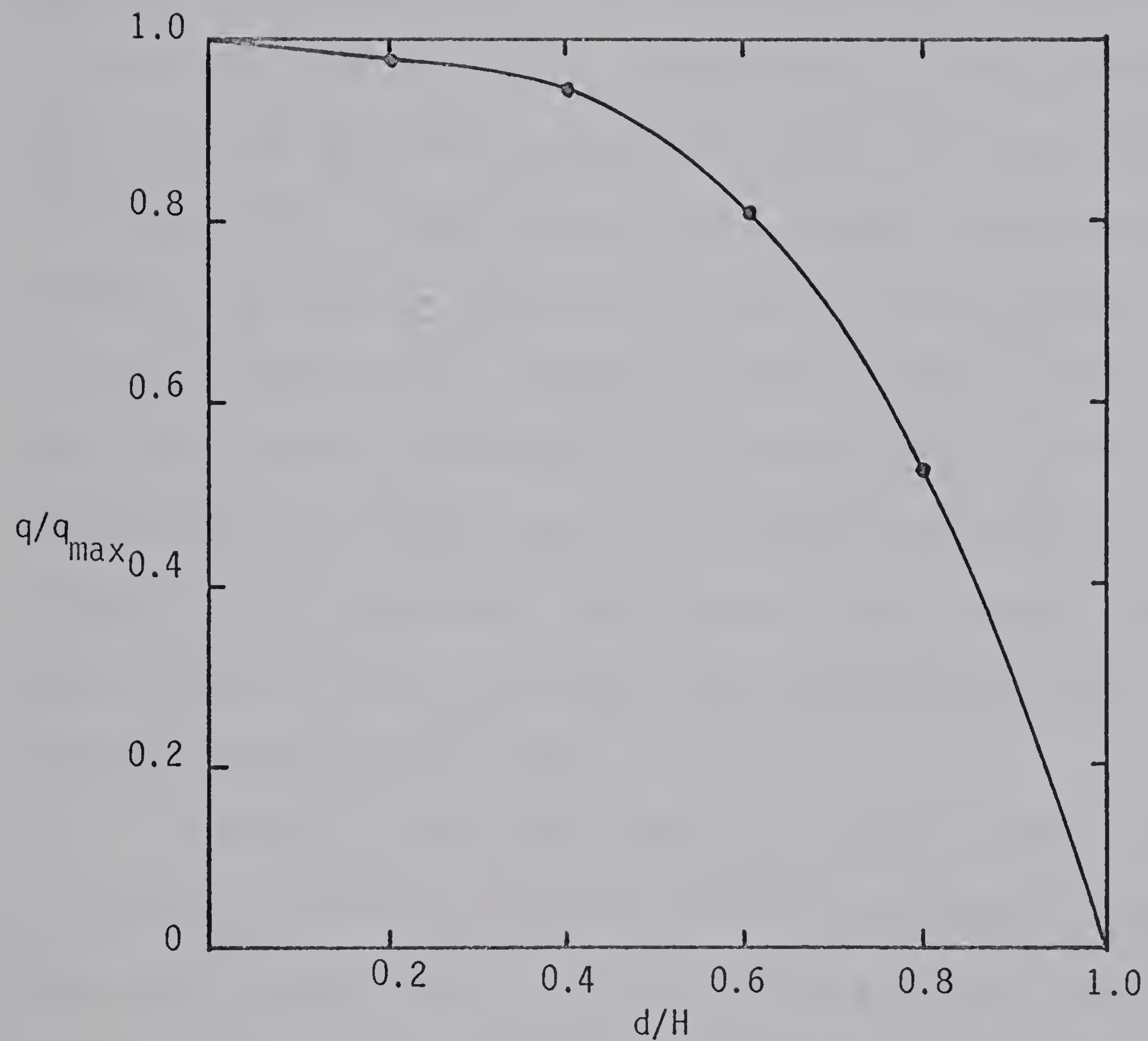


FIG. 3.5 FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

A plot between q/q_{max} and d/H is then developed and is shown on Fig. 3.6. In this plot q_{max} represents the discharge when there is no water in the reservoir, q is the discharge corresponding to the respective water level d in the reservoir and H is the head at the upstream boundary. As evident from this curve the relationship between the discharge and the reservoir water levels is highly non-linear. The reduction in discharge is not very significant at low reservoir water levels, while it becomes quite appreciable at higher levels, e.g., at d/h = 0.8, the reduction in discharge is almost 50%.



LEGEND:

q_{max} = Discharge corresponding to d=0
q = Discharge at particular reservoir level d
H = Head at the U/S boundary
d = Reservoir level

FIG. 3.6 PLOT OF DISCHARGE AS A FUNCTION OF RESERVOIR LEVEL

3.3 The Constant Discharge Concept

It is now well known that if the main sources of ground water recharge are precipitation, glaciers, influent streams, lakes and reservoirs, there exists a balance between inflow and outflow in the ground water basin under equilibrium conditions. This results in a certain amount of outflow from the banks, especially in hilly terrain. This outflow may vary seasonally but if all the sources of recharge have a cumulative effect on the occurrence and movement of ground water, the seasonal variations may not be very significant. However, there could be some cases when the seasonal variation of ground water table is quite appreciable. In that case the safest approach for stability analysis is to consider the highest and steepest ground water table. This would give the maximum outflow and obviously be the worst case.

In most of the field cases the local topography plays an important role in controlling the regional pattern of the ground water flow. As the filling of the reservoir proceeds the flow pattern is altered. Although the transient conditions will prevail for sometime, the final steady state conditions are more realistic for all but the more impermeable materials. If the head at the upstream boundary is assumed to be constant, it is obvious that the raising of the reservoir water level would result in the reduction of discharge from the banks into the reservoir. This conclusion does not seem to be reasonable as according to the

concept of a recharge and discharge cycle and the conservation of mass, the amount of water entering at the upstream boundary under normal conditions has to go out. Further, if there is a continuity of flow, the raising of reservoir water level which is changing the exit conditions of the ground water flow, induces a raising of the ground water level on the upstream side. This will ultimately become asymtotic to the original water table at infinite distance. This means that impounding results in the reduction of discharge temporarily, corresponding to the transient flow conditions, but under steady state seepage when the ground water table is stabilized again, the amount of water flowing out should remain the same. Since the location of the upstream boundary is not changed, the permeability of the material is also assumed to be constant, the only factor controlling the amount of outflow at various reservoir water levels is the head at the upstream boundary. Hence the assumption of constant head is not valid for most of the field cases; on the contrary, the concept of constant discharge seems to be more reasonable.

To evaluate the effects of this hypothesis, the same problem of a hypothetical slope with typical boundary conditions as explained in Section 3.2 is studied. The free surface line and the potentials for the case when the reservoir is empty is determined (Fig. 3.1) and by sketching flownets the amount of outflow from the slope into the reservoir is calculated (see Appendix B).

The increase in head required at the upstream boundary to keep the discharge constant at various reservoir water levels is then found. No simple relationship could be established due to the non-linear relationship between the reduction in discharge and the increase in reservoir water level. As an alternative the method of trial and error is followed and the head required at the upstream boundary to keep the discharge constant at various reservoir water levels is then determined.

A dimensionless curve, which is a plot between H_{req}/H_{max} and d/H_{max} for a constant discharge is developed and presented on Fig. 3.7. In this plot H_{max} is the initial head at the upstream boundary corresponding to the condition when there is no water in the reservoir. H_{req} represents the head required at the upstream boundary to give the same discharge at various reservoir water levels d. The line of constant discharge shows that this relationship is also highly non-linear. It may be seen further that the increase in head required at low reservoir water levels is not very significant, whereas at the higher water levels the increase in head required is quite appreciable, e.g., for ratios of d/H_{max} from 0.8 to 1.0, the increase in head required ranges from 20 to 40% (Fig. 3.7).

This curve may be used for determining the head required at various reservoir water levels for a typical value of q/kH_{max}. From this curve the head required at the upstream boundary for various reservoir water levels is

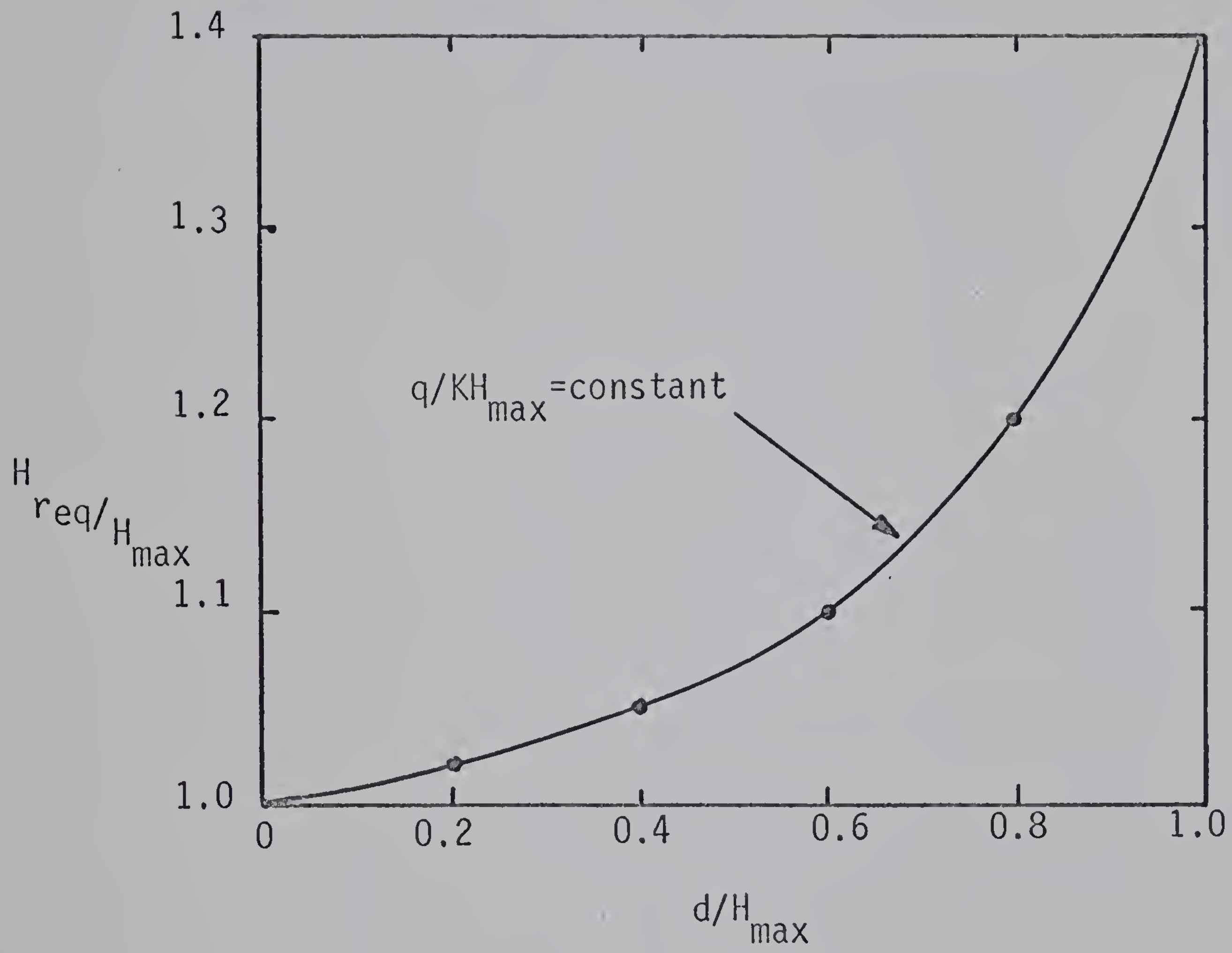


FIG. 3.7 DIMENSIONLESS PLOT OF HEAD REQUIRED FOR DIFFERENT RESERVOIR LEVELS

determined and then the free surface and potentials are obtained for the corresponding reservoir levels ranging from 400 to 1000 in increments of 200. As the increase in head at 200 elevation of the reservoir water level is very small, this case has not been studied.

These plots of free surface and equipotentials are shown on Fig. 3.8 through 3.11.

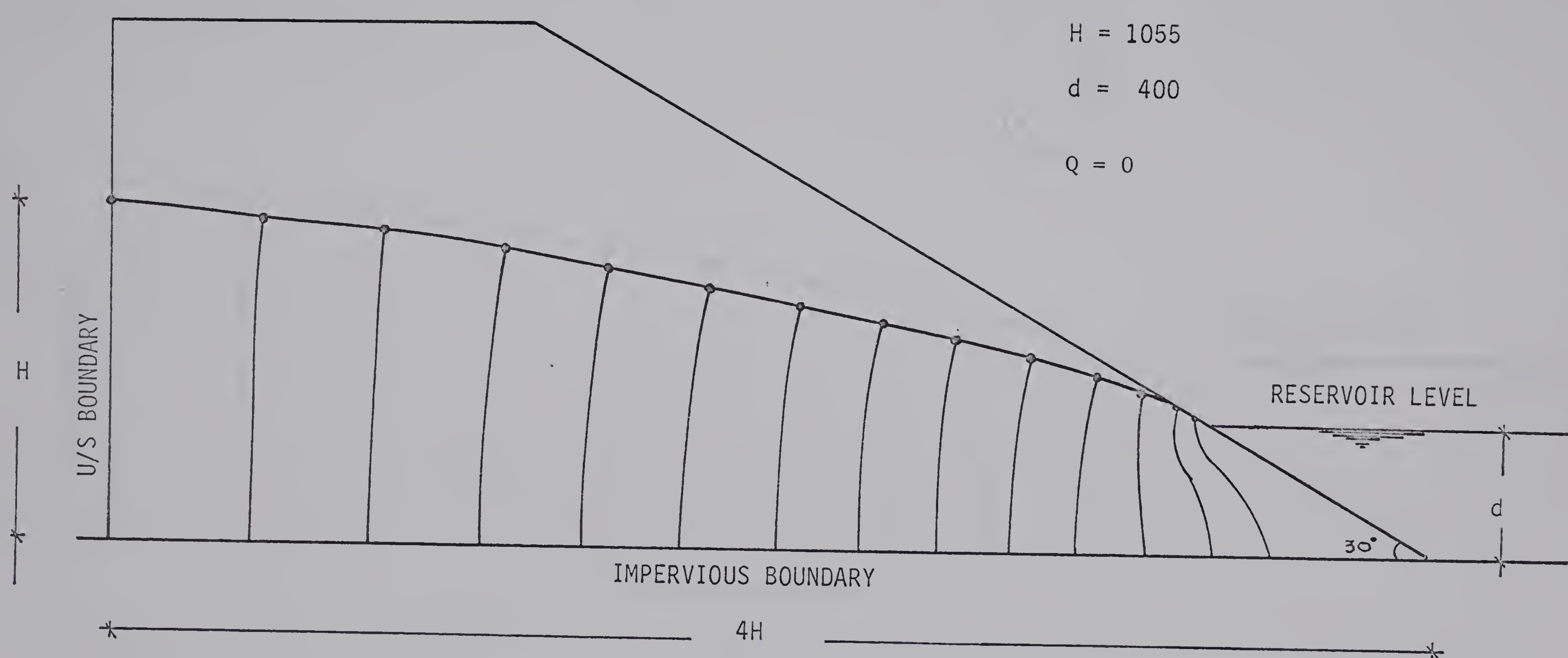


FIG. 3.8 FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

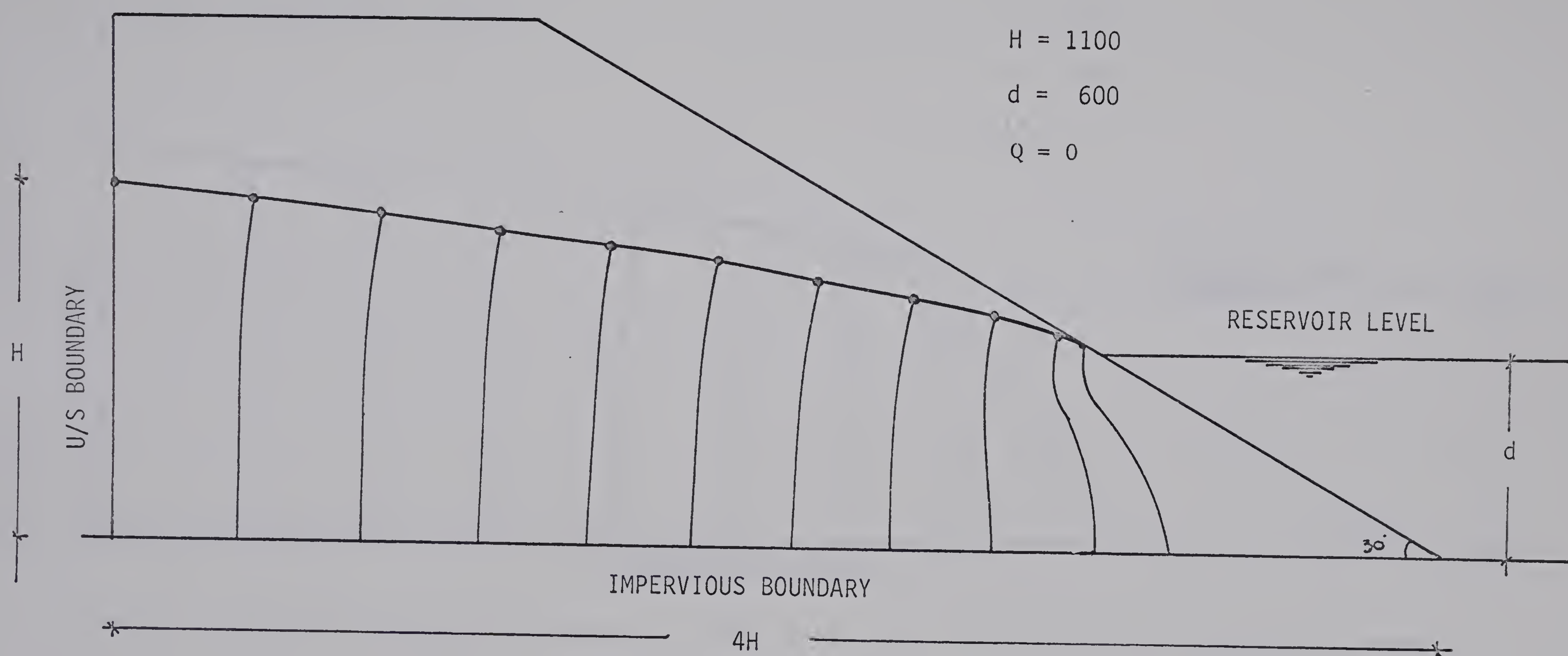


FIG. 3.9 FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

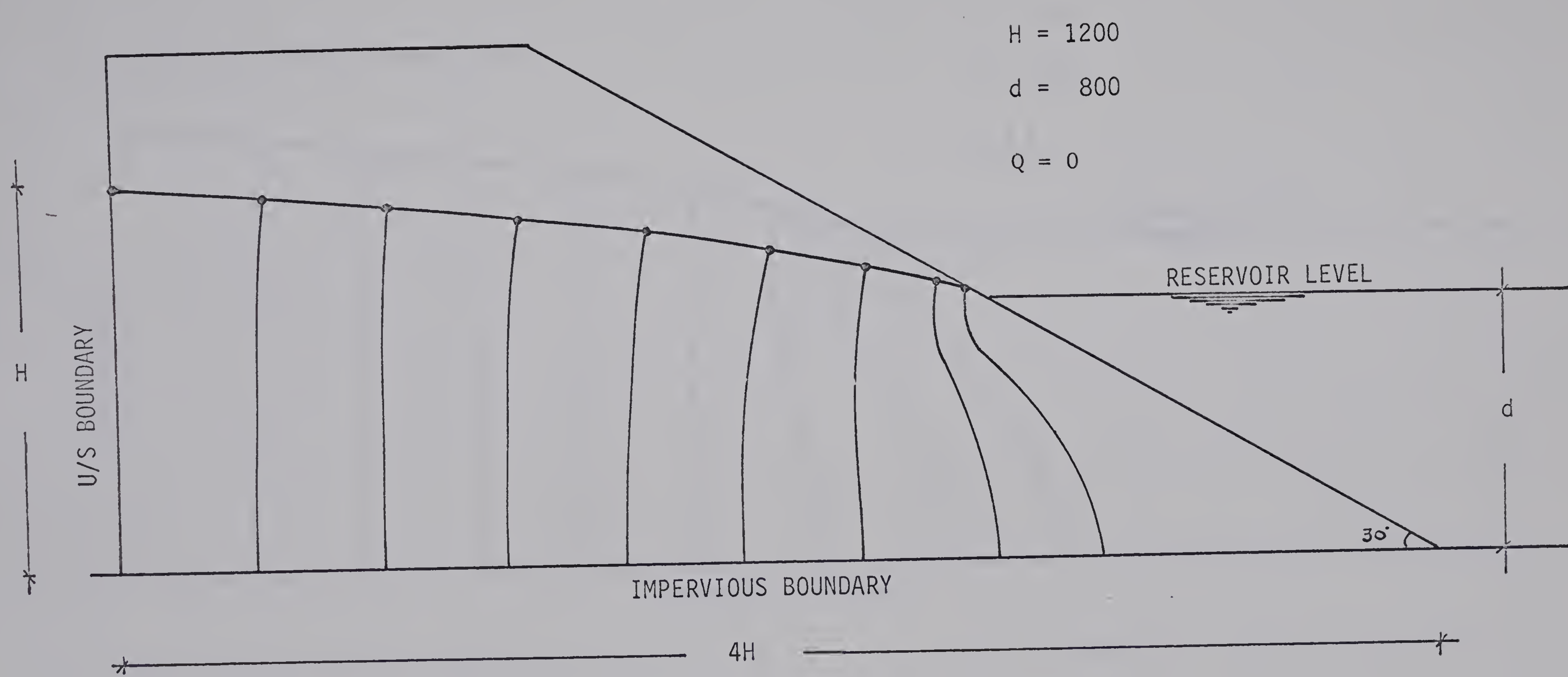


FIG. 3.10 FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

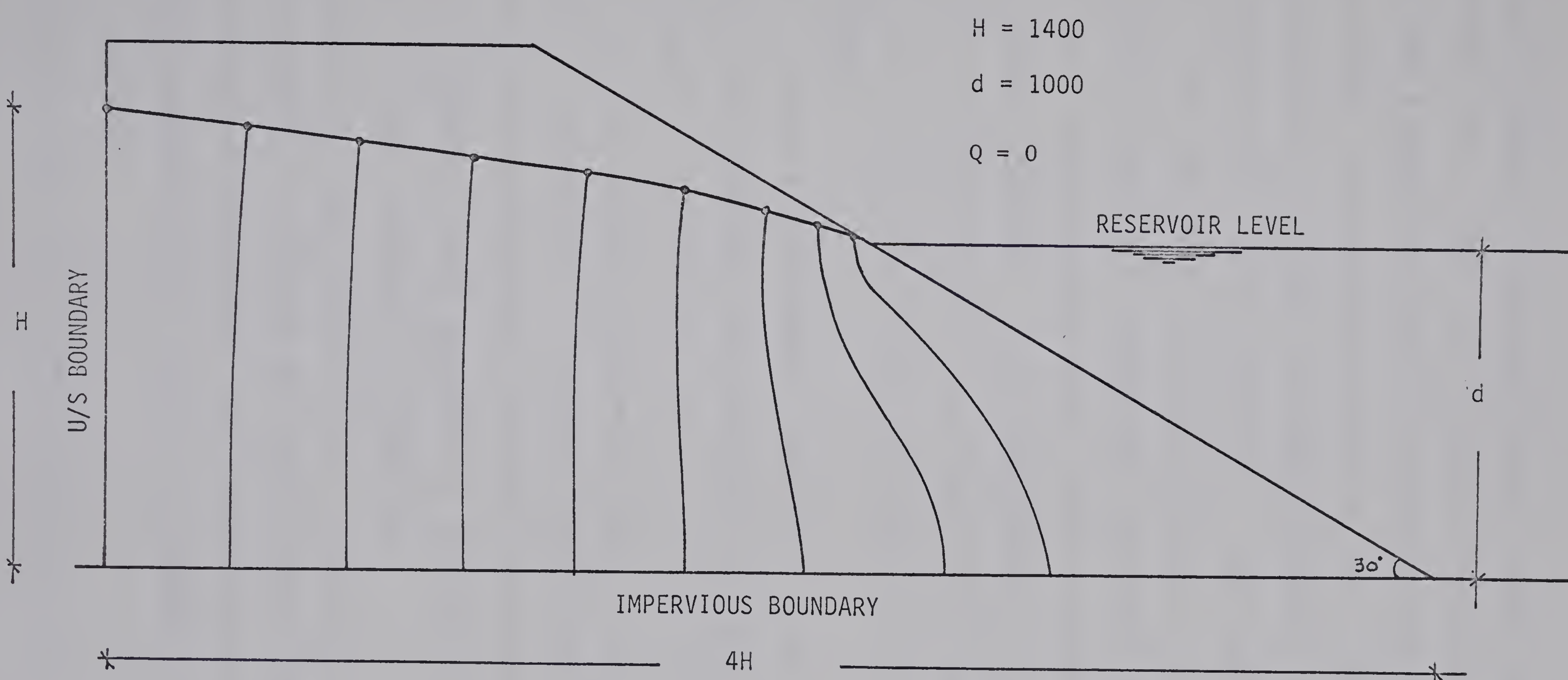


FIG. 3.11 FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

3.4 Effect of Changes in Reservoir Level on the Stability

Ignoring the Terzaghi and Skempton softening mechanism, almost all the studies dealing with the long term stability of natural slopes and embankment based on slip circle analysis show that the factor of safety increases with the raising of water level in the reservoir. The horizontal component of the hydrostatic pressure obtained as a result of the filling of reservoir stabilizes the slopes and as such with the raising of water level in the reservoir the factor of safety increases. This observation is however based on the assumption that the raising of the water level in the reservoir does not influence the head at the upstream boundary.

It is already established in Section 3.3 that the impounding of the reservoir does influence the head at the upstream boundary and therefore, it is not necessary that the factor of safety should improve at higher reservoir water levels. The increase in head at the upstream boundary corresponding to the respective reservoir water levels results in an increase of pore pressure in the soil or rock mass, which in turn, reduces the stability.

To assess the implications of this hypothesis on the stability of slopes, a typical 30° slope with an impermeable base, consisting of homogeneous and isotropic material having strength parameters of $\phi' = 40°$, $c' = 50t/m^2$, $\gamma_t = 2.2t/m^3$ is analyzed.

As shown on Fig. 3.12 it may be seen that the factor

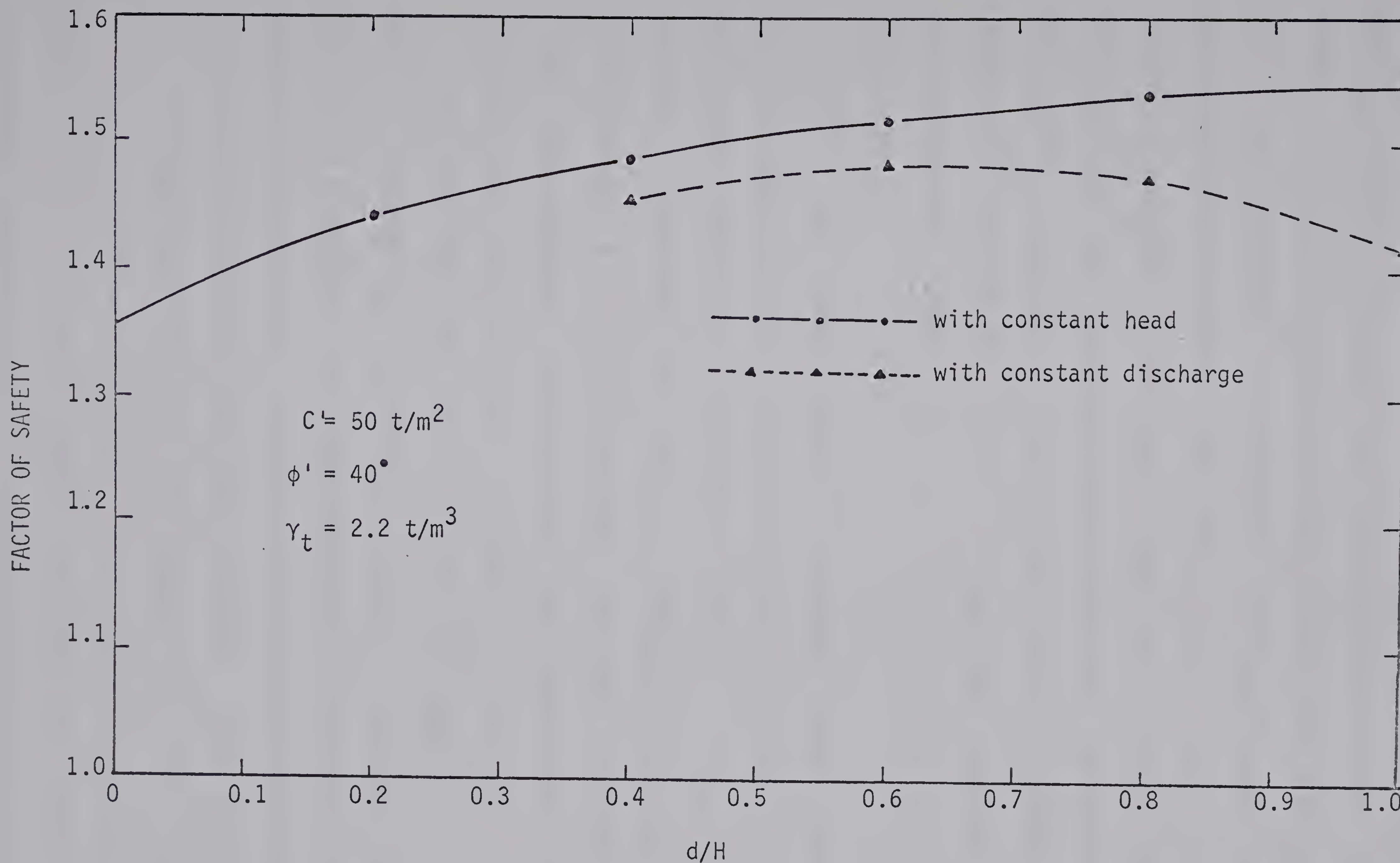


FIG. 3.12 VARIATION OF FACTOR OF SAFETY WITH RESERVOIR LEVELS

of safety obtained by using the Bishop simplified method (Bishop (1955)) increases with the raising of water level in the reservoir, provided the head at the upstream boundary is kept constant. If the head at the upstream boundary is adjusted in accordance with the reservoir level in order to keep discharge constant and if the pore pressure data thus obtained with these boundary conditions are used in the stability analysis, it may be seen that the factor of safety increases up to a reservoir level 650 and then starts decreasing with further raising of the reservoir level (Fig. 3.12).

It is also evident that this effect is more significant at higher reservoir levels, which is in contrast to most of the existing conventional studies. A reduction of about 9% in the factor of safety is witnessed when the reservoir level is at 1000 elevation. At lower reservoir levels the reduction is however not very significant but may be significant if the slope is already in a limiting state.

To illustrate the importance of this reduction in factor of safety more explicitly a case of reservoir level elevation at 1000.0 is selected and using different strength parameters a comparison of factor of safety obtained with the two hydraulic boundary conditions is made. Three values of angle of shearing resistance (ϕ') namely 40°, 30° and 20° are selected and for each value of ϕ', three values of cohesion (c'), i.e., $50 t/m^2$, $25 t/m^2$ and $5 t/m^2$ are tried. A relationship between factor of safety and angle of shearing

resistance is obtained for different values of c' and curves are developed using both the constant head and the constant discharge methods (Fig. 3.13 through 3.15).

As evident from Fig. 3.13 a typical slope of soil or rock mass having strength parameters c' = $50t/m^2$ and ϕ' = 29° would be considered as marginally stable if the factor of safety is obtained by using the constant head condition, whereas on the basis of the constant discharge concept, the slope would fail.

Similarly, referring to Fig. 3.14 and Fig. 3.15 slopes having strength parameters as c' = $25t/m^2$, ϕ' = 31° and c' = $5t/m^2$, ϕ' = 33 1/2° respectively may be treated as marginally safe under constant head conditions but are unstable when the constant discharge conditions are invoked. This obviously shows that even if the reduction in factor of safety is small, in the case of slopes of marginal stability, the effect of the constant discharge concept might be found very crucial.

Studies have already shown that when dealing with the long term stability of slopes, especially of fissured clay or rocks, a reduction in cohesion takes place due to a softening mechanism, which may be aggravated by the submergence of the slopes (Skempton (1964); DeLory (1957)).

If this phenomenon is taken into account the factor of safety for a 30° slope having values of c' and ϕ' as $50t/m^2$ and 33 1/2° respectively obtained as 1.25. Assuming the value of c' is reduced to $5t/m^2$ in due course of time,

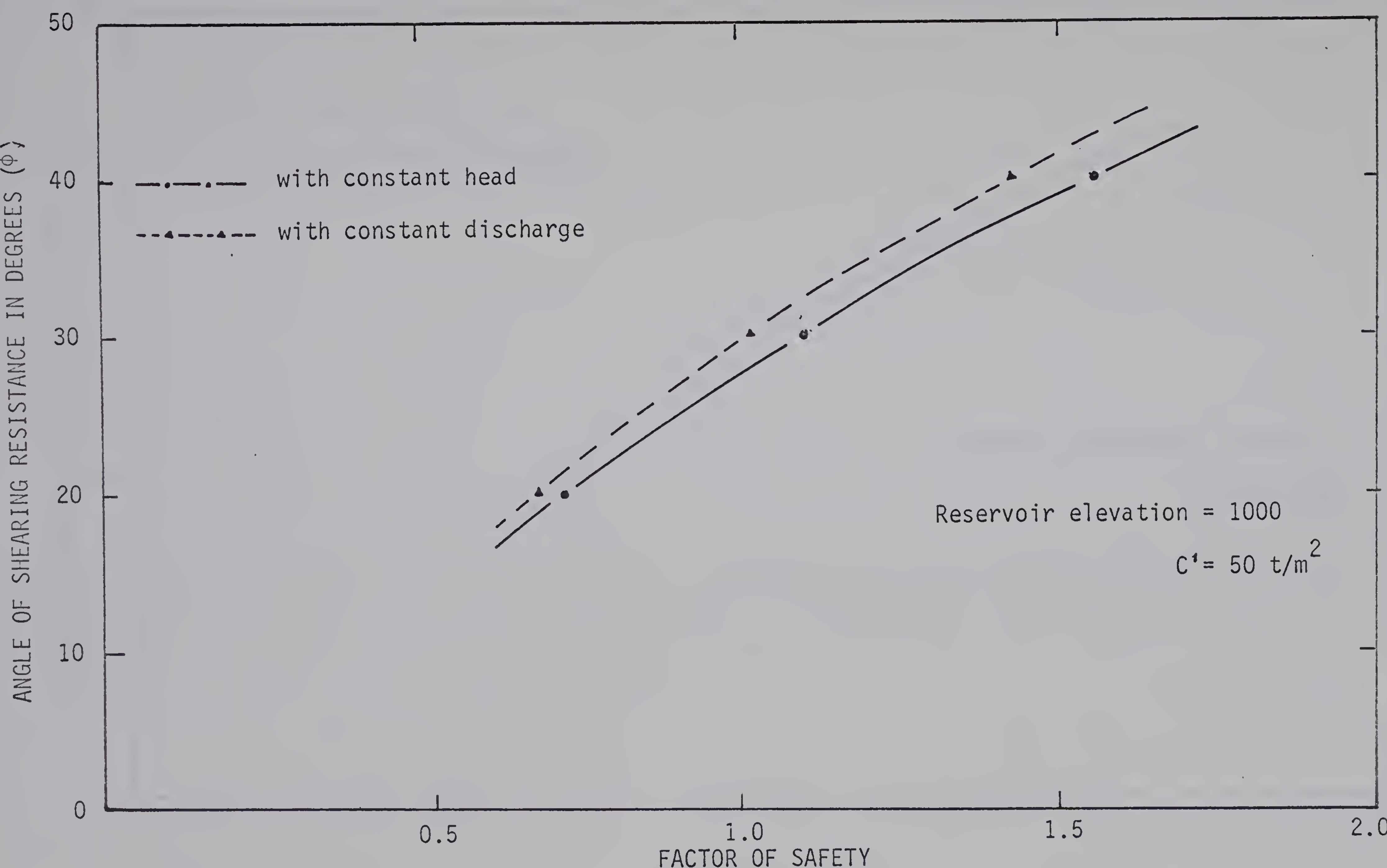


FIG. 3.13 RELATIONSHIP BETWEEN FACTOR OF SAFETY AND ANGLE OF SHEARING RESISTANCE

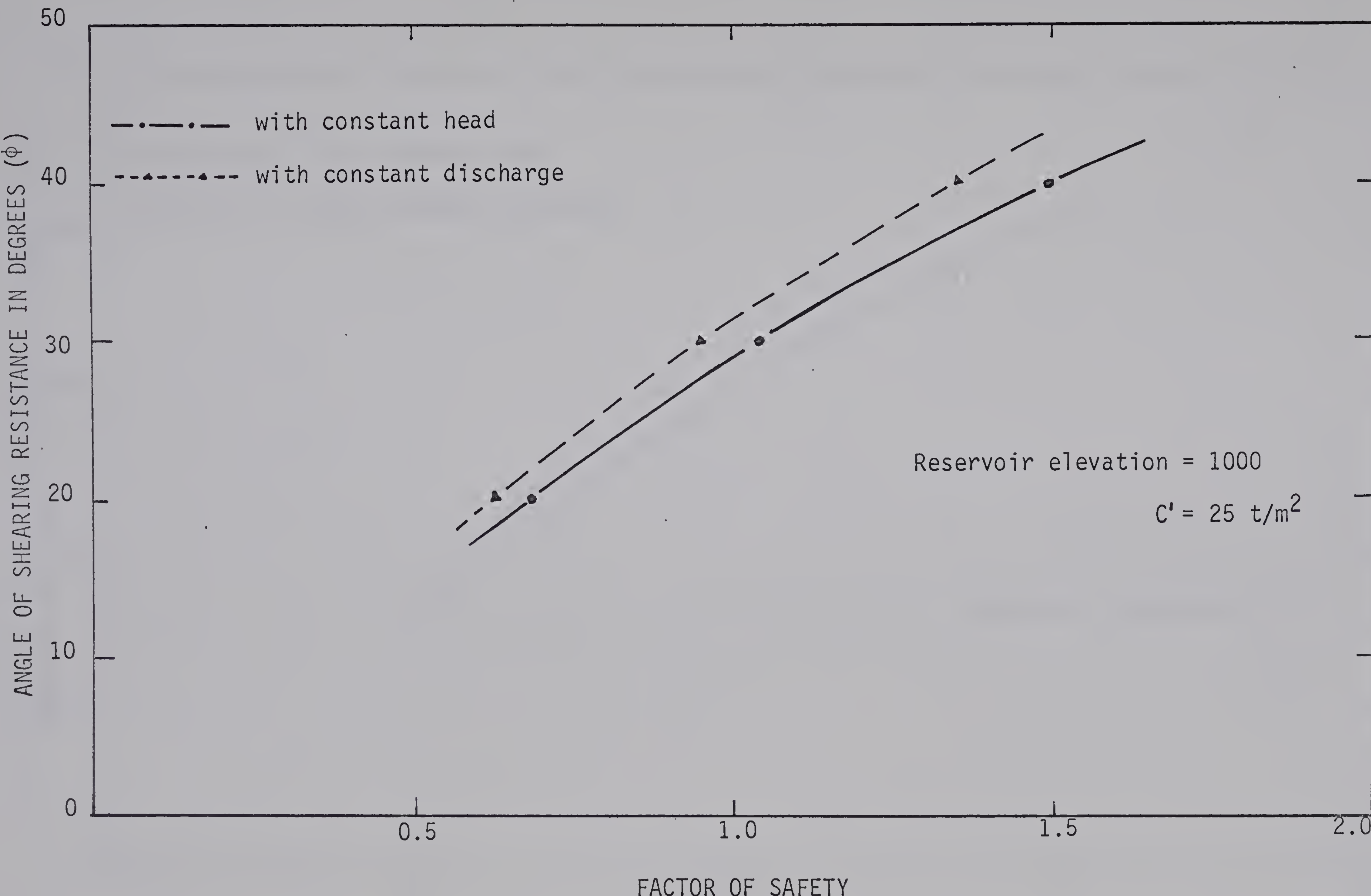


FIG. 3.14 RELATIONSHIP BETWEEN FACTOR OF SAFETY AND ANGLE OF SHEARING RESISTANCE

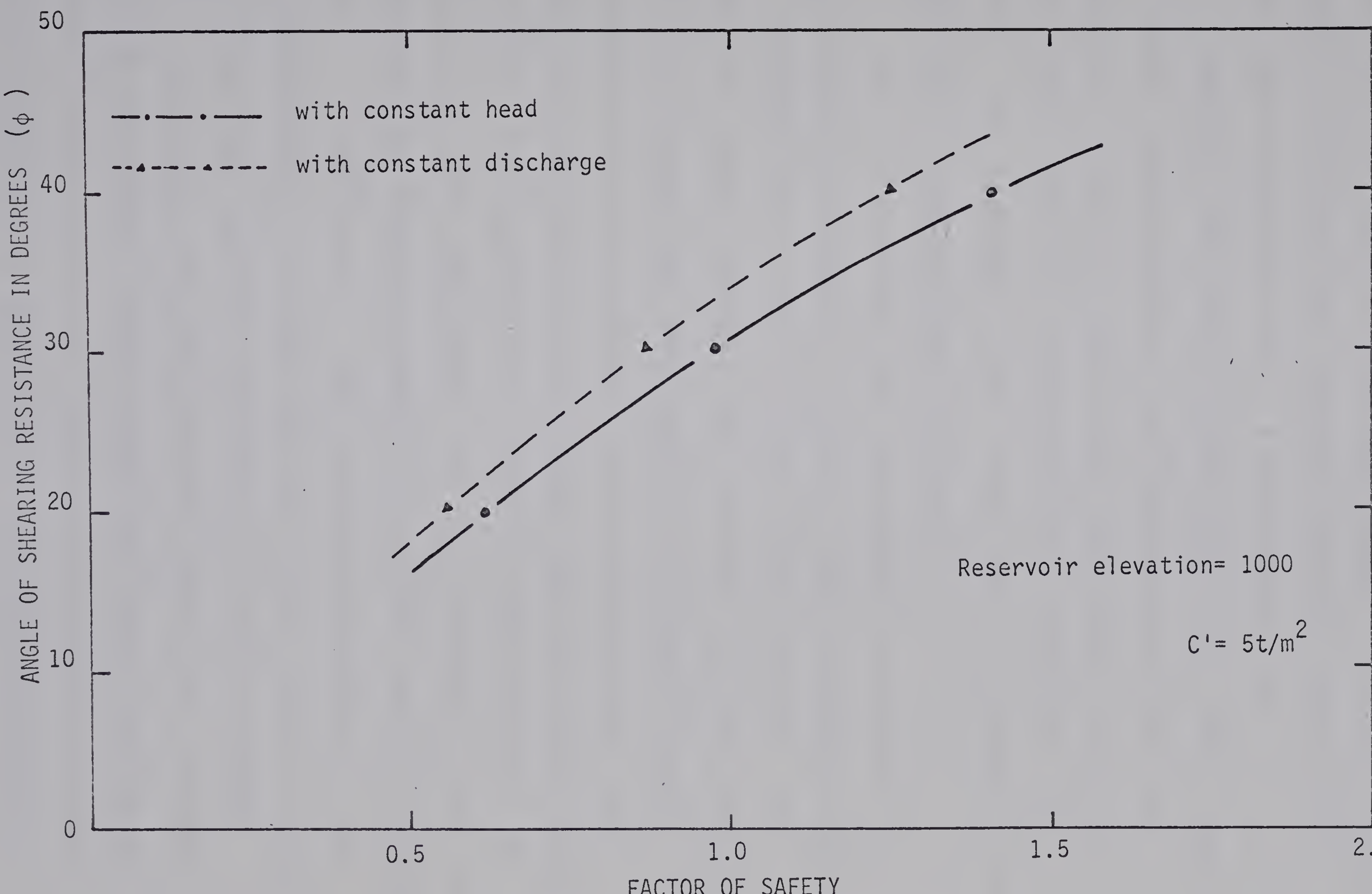


FIG. 3.15 RELATIONSHIP BETWEEN FACTOR OF SAFETY AND ANGLE OF SHEARING RESISTANCE

it may be seen that the slope is found to be marginally safe with the constant head condition but if the stability analysis is based on the constant discharge concept, the same slope is unstable (Fig. 3.13 through 3.15).

However, other things being equal, i.e., the c' and ϕ' values remain constant, the impounding results in an increase in factor of safety when slip surface is an arc of a circle, regardless of the discharge conditions (Fig. 3.12). This increase is less in the case of constant discharge than that of constant head conditions.

It is also evident that when there is no water in the reservoir a factor of safety of 1.358 is obtained with $c' = 50t/m^2$ and $\phi' = 40°$. As the filling proceeds, the factor of safety increases. At the 1000 reservoir elevation if it is assumed that the cohesion is reduced to $5t/m^2$, the factor of safety obtained by the constant head conditions is 1.411 which is still higher than the initial factor of safety corresponding to no water in the reservoir. On the contrary, if the constant discharge concept is used for determining the factor of safety at the 1000 reservoir elevation with $c' = 5t/m^2$ and $\phi' = 40°$, a factor of safety of 1.246 is obtained which is lower than the initial factor of safety and thus shows that this condition may trigger a slide.

It is also interesting to note that in this typical example of a slope, extreme conditions are not considered; that is to say the initial slope of the ground water table before impounding is about 1 in 5, the upstream head is

taken at an elevation of 1000 and its location is chosen at a distance four times the head inside the slope. There could be more unfavourable conditions which one can encounter in the field. As an example it may be quoted that depending upon the geohydrological conditions of the region and the local topography, the initial slope of the water table may be still steeper. This would simply mean that the amount of outflow from the banks would be more, which in turn, results in a further increase in head required at the upstream boundary for various reservoir levels. In that case a further increase in pore pressure due to these hydraulic boundary conditions would make the reduction of factor of safety much more significant.

It may further be noted that if non-homogeneity of the soil or rock mass is considered in conjunction with the constant discharge conditions, the initial head at the u/s boundary may further rise, resulting in higher pore pressures and thus a further reduction in the factor of safety. No detailed analysis for non-homogeneous material has been done, yet qualitatively the effect of non-homogeneity can be understood from a hypothetical model shown on Fig. 3.16.

As shown on this figure if the flow is occurring from low to high permeability material, since the discharge and the total head at the interface is the same, the free surface line in the less permeable material will be more steeper. As soon as it enters the more permeable material,

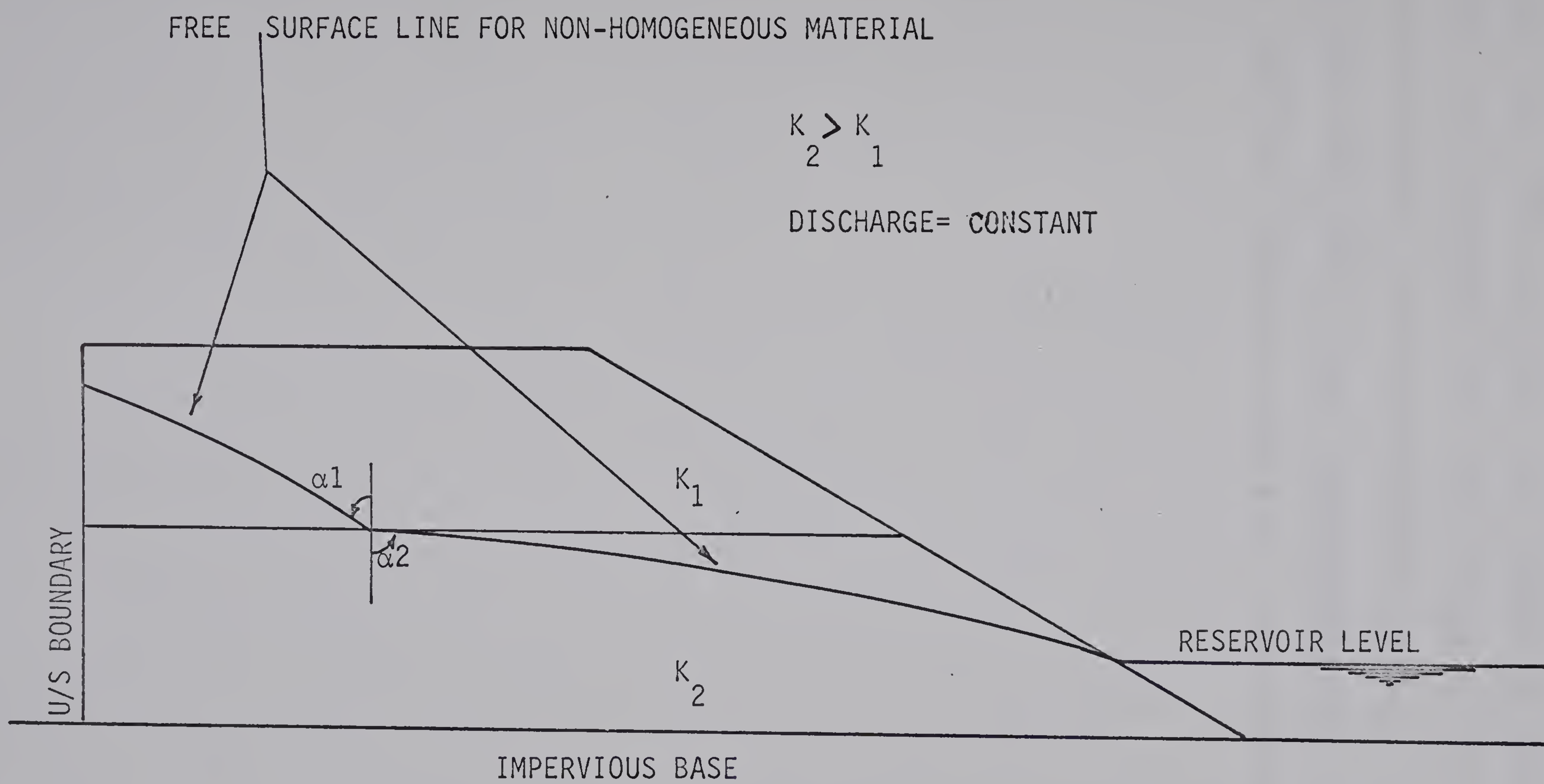


FIG. 3.16 HYPOTHETICAL MODEL SHOWING INFLUENCE OF NON-HOMOGENEITY ON THE FREE SURFACE

the free surface line becomes flatter and if the difference in the two permeabilities is large, the flow in the lower layer near the interface will almost be horizontal. Therefore, it is evident that under these conditions at the chosen location of the upstream boundary, the head will further rise due to this non-homogeneity, producing thereby an additional adverse effect on the stability.

3.5 Influence of Infiltration on the Stability

To evaluate the influence of infiltration on the stability of slopes, a constant infiltration of 5 cm/day over the entire free surface is taken. A dimensionless infiltration factor (Q) is introduced which is defined as the volume of water infiltrating per unit horizontal area per unit time divided by the coefficient of permeability of the soil or rock mass (i.e., $Q = - q/k$). The coefficient of permeability is taken as 100 cm/day, i.e., 1.1×10^{-3} cms/sec. The computer program is used to determine the free surface and potentials for the case when the reservoir level is at 800 (Fig. 3.18). The change in the free surface due to this infiltration is shown in Fig 3.17 and Fig. 3.18. As evident this results in the raising of the free surface line. The pore pressure data obtained from these flow conditions is then used in the stability analysis.

The case of upstream head at 1200 elevation and reservoir level at 800 with $c' = 50 t/m^2$, $\phi' = 40°$ and no infiltration over the free surface gives a factor of safety of 1.471. When a constant infiltration of 5 cm/day is introduced and assuming the coefficient of permeability as 1.1×10^{-3} cm/sec, the factor of safety obtained is 1.439. This obviously shows that the influence of direct infiltration on the increase in pore pressures and thus on the stability of slopes is not very significant. This is only true when the effect of infiltration is considered in terms

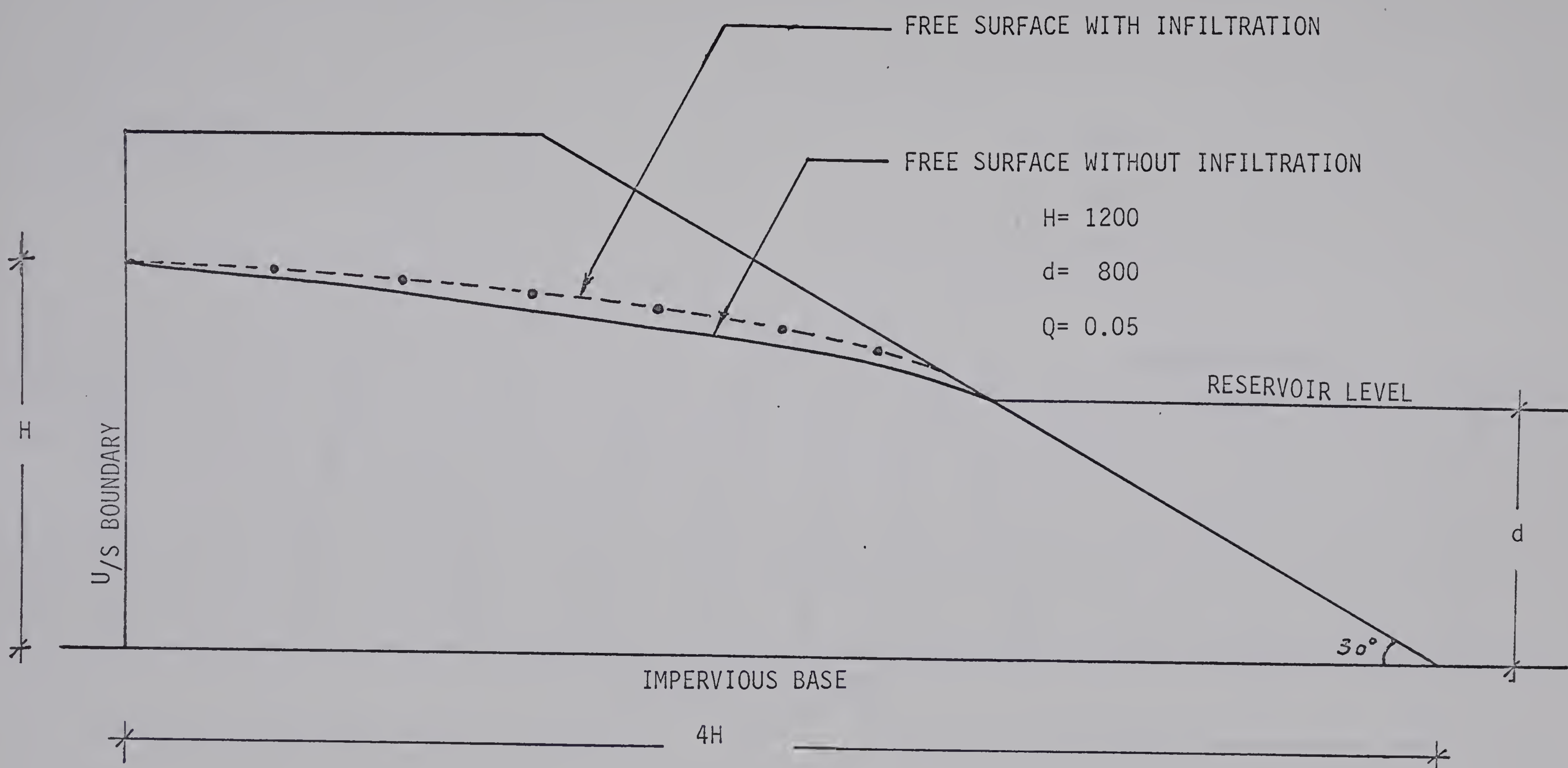


FIG. 3.17 DIAGRAM SHOWING INFLUENCE OF INFILTRATION ON THE FREE SURFACE

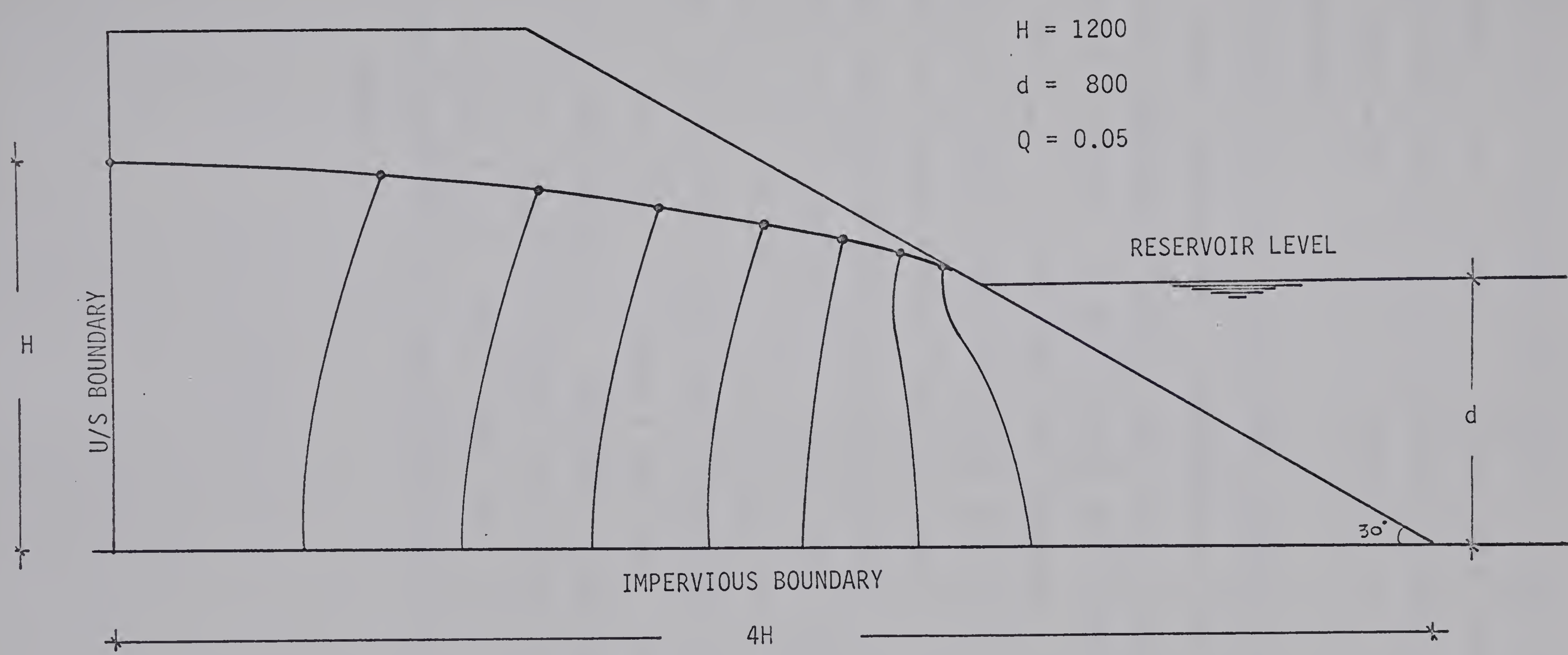


FIG. 3.18 FREE SURFACE AND EQUIPOTENTIALS FOR TYPICAL SLOPE (30°)

of raising of free water table only which in turn results in the increase of pore pressures in the stability analysis. No effect of infiltration on the filling of cracks resulting in the joint water pressure or softening etc. is taken into consideration.

Infiltration over the entire free surface as considered here, is a direct consequence of precipitation which depends not only on the geographic location but on the permeability of the material as well. It may be seen that with even more favourable values of run off factor and permeability an infiltration of 5 cm/day over the entire free surface may only be obtained from very high values of precipitation.

It may therefore be concluded that barring very extreme conditions of precipitation, direct infiltration on the free surface will result in the raising of the free surface line. This rise in the free ground water table due to infiltration does not induce a very significant in-increase in the pore pressures. Therefore, ignoring the effect of filling of cracks by water and softening of material above the free ground water table, this increase in pore pressures hardly produces any significant effect on the stability of slopes.

CHAPTER IV

DISCUSSION OF RESULTS AND CONCLUSIONS

The present work has been carried out with the object of evaluating the effect of changes in reservoir level on the stability of natural slopes. Although a detailed account of the results obtained is given in the preceding chapters, a brief summary of the results and conclusions drawn is given in the following paragraphs.

(1) A Fortran IV computer program using finite difference techniques has been developed which can be used for determining the steady state free surface potential flow through porous media. The program is quite versatile and can be used for any geometric configuration and boundary conditions. As a check, the numerical solution obtained by using this program is compared with Numerov's analytical solution of free surface seepage through a typical dam (Fig. 1.2). A close agreement between the two results is obtained. A value of 1.6 for the over-relaxation factor ensured the quickest convergence. This program can also be used for solving free surface problems with constant infiltration. This has been used in determining the free surface and potentials when a typical constant infiltration is introduced over the entire free surface.

(2) A study for the outflow from the banks at various reservoir levels shows that the relationship between the

reservoir level and the discharge is highly non-linear. At lower reservoir levels (up to 400 elevation), the reduction in discharge is about 6%, but at higher levels, e.g., at 800 elevation, the reduction in discharge is almost 50% (Fig. 3.6). This is however based on the assumption that the head at the upstream boundary remains constant.

(3) A hypothetical concept of constant discharge is postulated which results in a raising of head at the upstream boundary with the raising of reservoir level.

The occurrence and movement of ground water is an important phase of the hydrological cycle which always maintains a balance of inflow and outflow in the ground water basin. Any change in the exit conditions at the face of the slope which may correspond to the filling of reservoir need not result in the reduction of discharge from the bank especially in the areas of high relief. This obviously means that the assumption of constant head at the upstream boundary which is generally considered to be uninfluenced by the geometric configuration and/or the exit conditions of the slope is at variance with reality. It is therefore concluded that in defining the flow domain under steady state seepage conditions the concept of constant discharge which results in an increase in head at the upstream boundary with the raising of reservoir level is more realistic for field cases.

The transient conditions would always be on the safe side as far as the stability of slopes is concerned.

It is observed that the relationship between the head required at the upstream boundary to keep the discharge constant and the reservoir level is also non-linear (Fig. 3.7). At lower reservoir levels the increase is not very significant but at higher levels the increase in head required is quite appreciable, e.g., at 1000 reservoir elevation, the increase is 40%. Quantitatively this increase in head at the upstream boundary due to raising of reservoir level results in a raising of the free surface line and thereby an increase in the pore pressures in the stability analyses.

(4) The stability of a typical 30° slope consisting of a homogeneous, isotropic material and having an impermeable base is analyzed at various reservoir levels ranging from 0 to 1000 elevation in increments of 200.

The effect of impounding of the reservoir on the stability of slopes is then compared by using the constant head and the constant discharge methods. The material properties of $c' = 50 t/m^2$, $\phi' = 40°$ and $\gamma_t = 2.2 t/m^3$ are used in the Bishop simplified method for analyzing the stability of this typical slope. A minimum factor of safety is obtained in each case corresponding to various reservoir levels and hydraulic boundary conditions at the upstream boundary. They are given in Table 4.1 and Fig. 3.12.

It is observed that the factor of safety increases with the raising of reservoir level under constant head conditions. The factor of safety increases up to a reservoir

TABLE 4.1

VARIATION OF FACTOR OF SAFETY WITH RESERVOIR LEVEL AND UPSTREAM BOUNDARY HEAD

$c' = 50 t/m^2$ $\phi' = 40°$

Head At The Upstream Boundary	Min. Factor of Safety for Reservoir Level					
	0	200	400	600	800	1000
1000	1.358	1.443	1.484	1.518	1.540	1.556
1055			1.453			
1100				1.483		
1200					1.471	
1400						1.424
1200 with Q = 0.05					1.439	

Q = Infiltration Factor

elevation at 650 and then starts decreasing under constant discharge conditions. A decrease in the factor of safety of about 9% is witnessed at reservoir elevation 1000 if the constant discharge conditions are invoked. However, the factor of safety still remains higher than the initial value of 1.358 corresponding to no water in the reservoir. This shows that in this particular case the filling of reservoir produces a stabilizing effect even under the constant discharge conditions.

It may, however, be noted that in this particular problem extreme conditions are not considered. The amount of initial outflow depending on the initial slope of the ground water table is such that at reservoir elevation 1000, a 40% increase in the head at the upstream boundary is required. Depending upon the geohydrological conditions and the local topography, there could be field cases when the initial slope of the ground water table is steeper. In that case if the constant discharge concept is used, the head required at the upstream boundary will follow a different relationship and obviously higher values will be obtained. This means the rise in the ground water table will be more which will induce higher pore pressures. Under these conditions it is possible that at higher reservoir levels the factor of safety may become less than the initial value, and thereby may initiate instability.

It is observed that in the cases of marginal stability the application of the constant discharge concept results

in instability (Fig. 3.13 through Fig. 3.15).

(5) To investigate further effects of the constant discharge concept, a case of the reservoir level at 1000 elevation and upstream head at 1400 elevation is studied in detail. Different combinations of c' and ϕ' (e.g., c' = 50t/m^2, 25t/m^2, 5t/m^2 and ϕ' = 40°, 30° 20°) have been tried and sets of curves showing the relationship between the factor of safety and the angle of shearing resistance are obtained (Fig. 3.13 through Fig. 3.15). The minimum factor of safety obtained by using the constant head and the constant discharge conditions for different combinations of c' and ϕ' are presented in Table 4.2.

It is observed that a typical slope with c' = 50t/m^2 and ϕ' = 40° has a factor of safety of 1.358 before the filling of the reservoir. Under the constant head conditions the factor of safety increases with the raising of the reservoir level even if the value of c' is reduced to 5t/m^2. A factor of safety of 1.411 is obtained for c' = 5t/m^2 and ϕ' = 40° under constant head conditions, which is still higher than the initial value of 1.358 (Table 4.2). If the constant discharge conditions are considered, it is observed that a factor of safety of 1.246 is obtained for c' = 5t/m^2 and ϕ' = 40°. This obviously shows that the factor of safety has dropped down from its initial value and thus may result in triggering a slide.

An overall conclusion can therefore be drawn that depending upon the geohydrological conditions, local topo-

TABLE 4.2

COMPARISON OF FACTOR OF SAFETY

Reservoir Elevation = 1000

ϕ' (Degrees)	$c' = 50 t/m^2$		$c' = 25 t/m^2$		$c' = 5 t/m^2$	
	F.S.(1)	F.S.(2)	F.S.(1)	F.S.(2)	F.S.(1)	F.S.(2)
40	1.556	1.424	1.491	1.349	1.411	1.246
30	1.10	1.022	1.047	0.953	0.977	0.87
20	0.727	0.687	0.682	0.627	0.623	0.560

F.S.(1) = Factor of safety with constant head.

F.S.(2) = Factor of safety with constant discharge.

Upstream Head = 1000 (under constant head conditions).

Upstream Head = 1400 (under constant discharge conditions).

graphy and the material properties a stable slope corresponding to no water in the reservoir, which is usually considered the worst case may become unstable at higher reservoir elevations if the constant discharge conditions are invoked. Therefore, it is important, especially when working in the areas of high relief, to have a good knowledge of the geohydrological conditions of the area, so that more realistic hydraulic boundary conditions may be adopted in defining the flow domain.

(6) The stability analyses are done only for homogeneous, isotropic material. Although no quantitative explanation is given, it is of interest to note that the effect of non-homogeneity in addition to the constant discharge conditions produces an even more adverse effect on the stability of slopes (Fig. 3.16).

(7) The effect of direct infiltration on the free surface and then on the pore pressures, which are used in the stability analysis is evaluated. A case of reservoir level at 800 elevation and upstream boundary head at 1200 elevation with $c' = 50t/m^2$ and $\phi' = 40°$ is investigated. The coefficient of permeability is taken as 1.1×10^{-3} cm/sec. It is found that in this typical case the effect of infiltration, considered only in terms of an increase of pore pressure affects only the third significant figure in the factor of safety.

CHAPTER V

POSSIBLE EXPLANATION OF VAJONT ROCK SLIDE

On the basis of the constant discharge concept, it is now possible to give a qualitative explanation of the sliding movements at Vajont which occurred during the first stage of filling. Muller (1964) has mentioned that a few springs were observed at the toe of the left slope in the Vajont gorge before filling the reservoir. The piezometric observations in two piezometers were started in August, 1961 and in the third piezometer in October, 1961 (Muller, 1964).

At piezometer (PZ_2) located at the farthest point from the face of the slope the observations which were started in October, 1961 showed a water level close to 700 m. The other two piezometers (PZ_1 and PZ_3) which were located close to the valley recorded water levels of about 630 m. The reservoir level was fluctuating around 590 m. at that time.

These observations establish the fact that at the outset the mountain water table was sloping towards the valley and there was an outflow from the slopes into the valley. As no piezometric observations during the first stage of filling are available, the slope of the mountain water table was not known at that time. During the first filling when the reservoir level reached elevation 650 m. accelerated

movements started. The reservoir level was then lowered and at elevation 600 m. the movement practically stopped. Until November, 1961 the lake water level was kept constant and there was practically no displacement. From the beginning of October, 1961 to the beginning of February, 1962, the water level was raised from 590 m. to 650 m. and the movement remained small. Rocks started to move again as soon as the water level rose above the level 650 m. These movements continued until the beginning of 1963, when a short slow down occurred as the reservoir level was again dropped from 700 m. to about 650 m. During the third filling from the beginning of April to the end of May, 1963, the lake level rose from 650 m. to 696 m. and the displacements were again found negligible. When the water rose again from 696 m. to 710 m. rock displacements became alarming and resulted in the final phase of the failure.

Muller (1968) has tried to explain this first sliding movement by considering an artesian thrust of about 100000 t/m at the base of the sliding mass. Although nothing was known about the permeability of the individual strata, Muller (1968) has assumed that the permeability of the strata lying under the slip surface (Dogger) was more than the strata above, which resulted in this hydrodynamic pressure with a vertical component, corresponding to a maximum height of water column of 150 m. He further explains that during the first movement the friction descreased from its peak value to the smaller residual values. Yet the reduced frictional

resistance increased again when a state of rest sets in, the contact material consolidated and when new movements started the old peak strength value (or value near to it) was prevailing.

If Muller's assumptions that the material underlying the slip surface was more permeable and that the artesian thrust caused the first sliding movement at the reservoir level 635 m. is accepted, during the second filling the same phenomenon should have occurred at the reservoir level 635 m. But no movements were observed until the reservoir elevation reached 650 m. His explanation of regaining strength when a state of rest sets in is at variance with reality.

Kenney (1967) is of the opinion that the reservoir slope must have been in a very delicate state of stability for a long period of time. Assuming the material as purely frictional, he has found that the factor of safety decreases with the filling of the reservoir up to 900 m. elevation. At 700 m. elevation the decrease is 5-10% while at 635 m. elevation the decrease is only 4%. Although this decrease was small it caused the rock to move. He also assumes that since the coefficient of sliding resistance was less than 0.4 (corresponding to $\phi' = 22°$) when the reservoir slope failed, a large portion of the sliding surface passed through the clay material rather than through limestone.

A critique of Kenny's explanation is given as follows: The assumption that the rock material was purely frictional

is rather debateable. Therefore it is not necessary that the coefficient of sliding resistance may decrease with the filling of reservoir as hypothetically postulated by him.

Moreover, even if it is assumed that the slope material is purely frictional and that the sliding surface passed through the clay layers which had residual strength, the first filling of reservoir up to 635 m. elevation resulted in the reduction of the coefficient of sliding resistance by about 4% and caused the rock slope to move. But during the second filling no movements were observed until the reservoir level reached the 650 m. elevation. Therefore, it is argued that if the material is considered frictional and was at residual state, the second filling of the reservoir at elevation 635 m. would have again given the previous value of the mobilized coefficient of the sliding resistance (i.e., 4% reduction in the initial coefficient of sliding resistance at 635 m. reservoir elevation). The movements should have then started at 635 m. elevation again and not at 650 m. elevation during the second stage of filling.

On the basis of the low value of angle of shearing resistance obtained at failure Kenney (1967) concluded that the slip surface passed through the clayey material which was at residual state. Muller (1968) contradicted this possibility and in support has given results of detailed geological investigations showing thin layers of clayey material only very sparsely present between the bedding

planes. Hence the possibility of the slip surface passing through these thin layers of clayey material is remote.

Jaeger (1969) has explained this first time slide by assuming low values of angle of shearing resistance along the upper strata. He assumes that during this period the factor of safety was very close to unity and when the water receded it became more than one. The disappearance of any movement during the second stage of filling up to the former maximum level of 652 m. is attributed to the remarkable process of "auto-stabilization".

It can be argued that no field evidence is available to prove the assumption that the angle of shearing resistance of the upper strata was lower than the lower strata. On the contrary it seems more reasonable to assume lower values of friction in the lower strata which was submerged. Moreover the process of auto-stabilization as given by him does not explain fully the reason as to why the displacements did not start again at the reservoir elevation 635 m. during the second stage of filling. If the material was at this residual state, the displacements should have started again at reservoir elevation 635 m. as no regain of strength is possible even by this process of auto-stabilization.

To summarize it may be stated that although many excellent references are available which describe the possible causes of the Vajont rock slide, none of them is free from unrealistic assumptions and unresolved questions.

In spite of the scanty information, the rock displace-

ments in the Vajont valley corresponding to the various reservoir levels can be explained qualitatively on the basis of the constant discharge concept.

It may be argued that before the beginning of the first filling, the mountain water table was steep. The initial factor of safety corresponding to the case of no impounding must have been more than unity. As the filling proceeded the water table must have risen upward in order to keep the discharge constant. Due to this rise in mountain water table pore pressure increased which in turn reduced the factor of safety very close to unity and the rock slopes started to move. During the period of lowering and then of constant reservoir level, the movement stopped as the water table came down and the factor of safety become more than one again. When the second filling started the rock slope did not move until the former reservoir elevation of 650 m. was crossed. This is quite evident from the fact that during the second filling the water table started rising again and as soon as it reached to the previous position corresponding to the reservoir level 650, the factor of safety became very close to unity and the rock slope started to move again.

It is of interest to note that the rock material which was submerged became more permeable and during the last stages of filling exhibited fully drained conditions. It is expected that the rock material in the beds stratified horizontally became buoyant due to submergence and as a

result of this stress release, the joints and fissures opened up. Hence the permeability of the rock material in these horizontal layers increased, whereas the permeability of the inclined layers of Malm and lower cretaceous formations which were not submerged remained the same. As the amount of water flowing out of the ground water basin is constant and the head loss in the less permeable material is more, it becomes necessary that the water table must have a steeper gradient in these inclined layers of comparatively low permeability. Hence it is concluded that due to this non-homogeneity the slope of the mountain water table in the rock beds stratified obliquely and horizontally was not the same.

With the filling of the reservoir, since the exit conditions are disturbed and the excess of water in the ground water basin has to get out, the effect of raising the reservoir on the water table in the less permeable material will be more adverse. The water table in the more permeable material will become flatter, while it will be steeper in the less permeable material hence inducing higher seepage forces.

No explanation has been given of the fact that during the first filling the movements started at reservoir elevation 635 m. and were very high at 650 m. elevation but that during the second filling the movements started at reservoir elevation 650 m. only. If the hypothesis of constant discharge is accepted it can be argued that since the filling

of the reservoir results in the raising of the water table, the movements should have started again at reservoir elevation 635 m. during the second stage of filling. This is explained by the simple fact that the first reservoir filling up to 635 m. and then to the 650 m. elevation was done during the wet period of the year (October, 1960 - November, 1960). The precipitation is generally high during these months in that area and as the recharge of the ground water basin will be more, it is obvious that the discharge passing out will be more as compared to the other periods of the year. This means that during this period of the first filling, the mountain water table was slightly higher and steeper, passing slightly higher discharges than in the normal periods. During the second stage of filling the reservoir level of 635 m. was reached during the colder month of January, 1961, which is a comparatively dry period and thus it is expected that during this period the mountain water table must be slightly lower than the previous case as the discharge passing out is less. Therefore, it is evident that at the reservoir elevation of 635 m. the same conditions did not occur again, the water table is still lower than the previous case and so the factor of safety is also slightly more. The factor of safety became close to unity again when the reservoir elevation was at 650 m. and thus the movements started again. The same phenomenon occurred again in 1963 during the second lowering and third filling. Moreover, as the permeability of the material which was

submerged earlier has increased the increase in head in the material of two different permeabilities material will not be the same as in the previous case. The slope of the mountain water table will become flatter if the permeability has increased, which means less seepage forces than the previous case.

Muller (1968) has also mentioned that although heavy precipitation occurred in the months of August and September 1963 prior to failure, no correlation with the piezometric observations could be established. This shows that some distinct changes in the permeability of the rock material along the inclined and horizontal beddings might have taken place.

Another question still remains unresolved. The piezometric observations during the later part of filling do not show any correlation between a raising of the mountain water table with the raising of the lake level.

This may be explained by the fact that during the first filling, no piezometric observations were made, as such the initial slope of the mountain water table was not known. As the submergence proceeded, the permeability of the submerged rock material increased considerably, while the inclined layers of Malm and lower cretaceous still had low permeability. It therefore seems necessary that the mountain water table should have a steeper gradient in these layers than the ones which were submerged. No data of the permeability as well as the mountain water table in the in-

clined layers of Malm and lower cretaceous is available.

However, it appears from the locations of these piezometers that they were observing the water levels in the more permeable material where the bedding was almost horizontal. Hence it is evident that these are not representative observations of the mountain water table in the less permeable inclined layers of Malm and lower cretaceous formation.

A hypothetical free surface line in these layers is shown on Fig. 5.1. The x-section is taken from Muller's report (1968) and gives the possible explanation for not observing higher mountain water table in this region. As shown the gradient of the water table in the inclinded layers is steep, resulting in high pore pressures and seepage forces, while the water table in the horizontal beds where the piezometers were installed is quite flat.

It is also interesting to note that depending upon the failure profile chosen, several investigators have found the values of angle of shearing resistance at failure ranging from 17.5° to 24° (Muller (1968)). Nonveiller (1967) has also found the values of angle of shearing resistance at failure from 27° to 28.5°, but the assumptions of slip surface position and form differ very much from nature (Muller (1968)). Kenney (1967) obtained a value of angle of shearing resistance less than 22° at failure. These values are much smaller than what could be expected for limestone.

LEGEND:

1= Malm
2= Upper-Malm-Lower-Cretaceous-formation
3= Lower-Cretaceous-formation
4= Lower-Cretaceous-formation
5= Lower-Cretaceous-(Albian)-formation
6= Upper-Cretaceous-(Cenoman-) formation
7= Upper-Cretaceous-(Cenoman-) formation
8= Upper-Cretaceous-(Tur.-,Senon.-) formation
PZ_2= Piezometer number 2

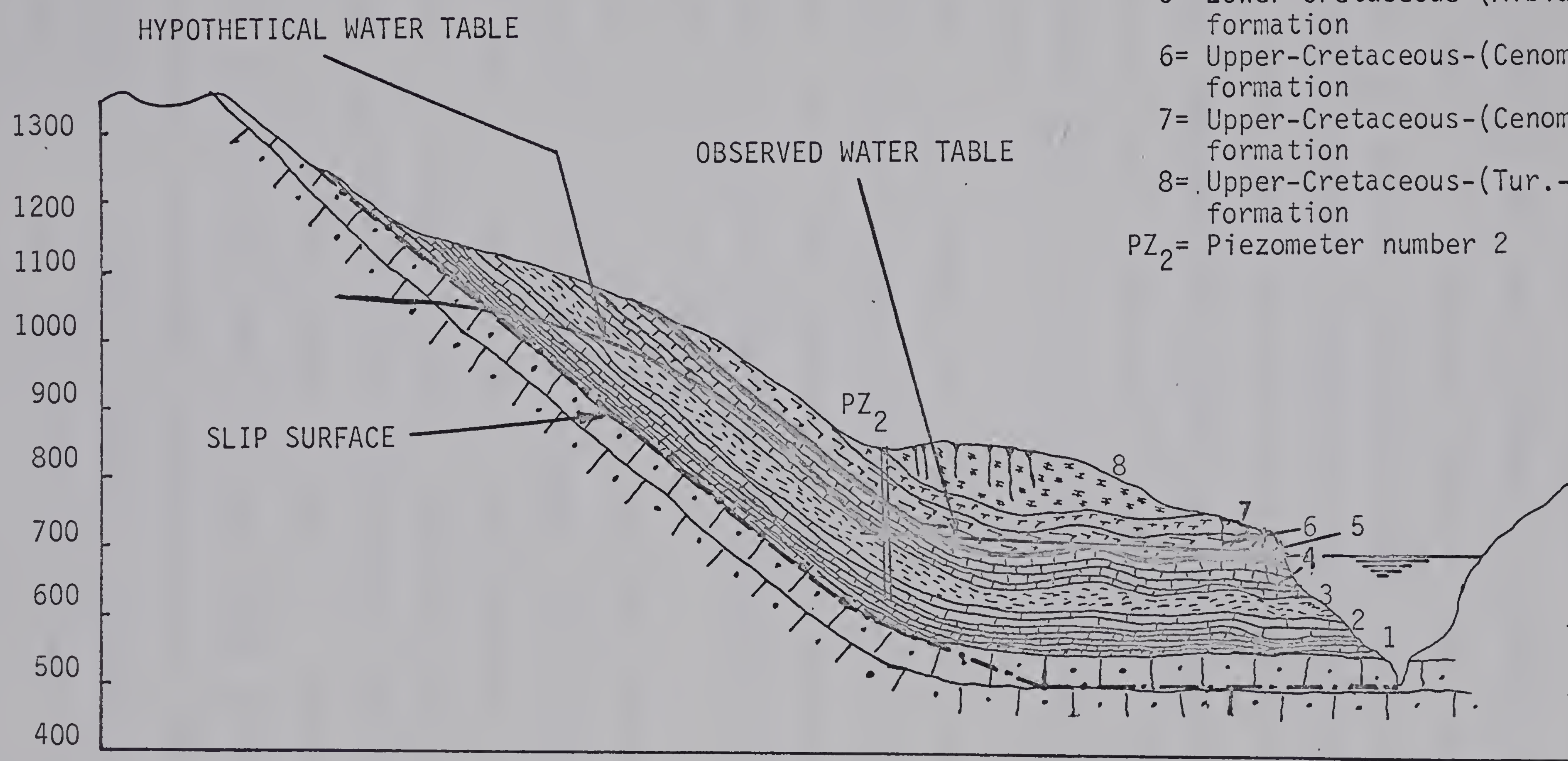


FIG. 5.1 DIAGRAM SHOWING GEOLOGICAL SECTION OF THE VAJONT ROCK SLIDE WITH HYPOTHETICAL WATER TABLE (GEOLOGICAL SECTION TAKEN FROM FIG. 2. MULLER(1968))

According to Skempton (1966) a friction angle for limestone of about 30° should be expected. He is also of the opinion that in the case of Vajont a reduction of friction, as a result of slip between adjacent beds should not be considered, as none of the observations showed any smoothing of the roughness pattern on the bedding plane.

It is now evident that these low values of friction which are not representative values for the limestone are obtained because in all the stability analyses, the water table is assumed more or less horizontal and equal to the reservoir level.

If high seepage forces as a result of a steeper water table in the inclined layers of Malm and lower cretaceous formations are taken into account, it seems necessary to obtain a value of angle of shearing resistance close to the representative value of limestone.

To check this, an approximate stability analysis based on Janbu's et al. (1956) method is done. The valley cross-section and the slip surface chosen correspond to slip surface II, Fig. 2 in Kenney's paper. The pore pressure inserted in the stability analysis are based on the hypothetical free surface line shown in Fig. 5.1. The stability calculations indicate that when the rock slope failed the average coefficient of sliding resistance was 0.554. This value correspond to an angle of shearing resistance of 29°, which is obviously a more representative value for limestone.

To summarize it is concluded that this possible explanation of the Vajont rock slide, based on the constant discharge concept, provides a qualitative answer to most of the unresolved questions pertaining to the complexities of this major catastrophe. At the same time it is realized that in the absence of any data on the permeabilities of different layers and the mountain water table in the inclined layers, this explanation is purely hypothetical. However, it may contribute to an understanding of the importance of the detailed knowledge of ground water hydrology and in-situ permeability in natural slope stability problems.

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APPENDIX A

- NOMENCLATURE FOR COMPUTER PROGRAM
- LISTING OF THE COMPUTER PROGRAM

Nomenclature for Computer Program

H( )	Grid point potential.
Hs( )	Potential at the free surface; 2nd and 3rd column of the same matrix stores interpolated values of potentials in the vertical direction corresponding to a particular point on the free surface.
X( )	Abscissa of the grid point.
Y( )	Ordinate of the grid point.
RES( )	Residue at each grid point.
CHK( )	Matrix which ensures convergence of the free surface potentials within the allowable limits of error.
KGP( )	Array which stores the specific free surface points which are coincident with the grid points.
NPP( )	Array which stores the actual grid points which may correspond to the free surface points.
N	Number of free surface points.
Q	Dimensionless infiltration factor.
DELTA	Mesh size.
I_{MAX}	Maximum number of rows in the H( ) matrix.
J_{MAX}	Maximum number of column in the H( ) matrix.
NE	Number of linear equations to be solved by subroutine SOLVE.
ARESM	Maximum allowable residue.

MITR	Maximum number of iterations required for convergence.
EPSLN	Over-relaxation factor.
ALERO	Allowable error in the free surface potentials.
RELAX	Name of the subroutine used for relaxation by SOR.
PINT	Name of the subroutine used for interpolation of grid potentials.
SOLVE	Name of the subroutine used for assigning values of coefficients and constants to be used in subroutines ARRA and SIMQ.
ARRA	Name of the subroutine which converts a matrix into an array (SSP).
SIMQ	Name of the subroutine used for solving linear equations simultaneously (SSP).
LCNT	Instantaneous count for the number of iterations in subroutine RELAX before achieving convergence.
KGPCH	Index used to trace the points in the array KGP.
LUT	Index for storing points for array KGP.
KAN( )	Array used for deciphering interpolation in X and Y directions.
YY	Variable used to reduce the ordinate values in steps of delta.
KQQ	Index for array KGP.
KA	Index for array KAN.

NL_1	Index used for a particular row in matrix H( ).
ML_1	Index used for a particular column in matrix Hs( ).
A( )	Matrix for coefficients used in the linear equations.
C( )	Array for storing constants of simultaneous linear equations.
ALPHA	Angle subtended by the segment of the free surface line with the horizontal.
$DELT_1$, $DELT_2$	Horizontal distances of grid points which are interpolated or otherwise used in the polynomial.
$DELT_3$, $DELT_4$	Vertical distances of the grid points which are interpolated or otherwise used in the polynomial.
JAR_1, JAR_2	Indices used for those surface potentials that are required to be used as constant in the solution of polynomial.
KAJ	Index used for specifying column number in the matrix H( ).
C(5)	Constant for infiltration factor.

MAIN

MAIN

```
C       INFILTRATION

        COMMON H(30,60),HS(60,3),X(60),Y(60),RES(500),CHK(60
       *,160),KGP(25),
       1NPP(60)
        COMMON N,Q,DELTA

C       INPUT DATA

        READ (5,10)IMAX,JMAX,N,NE,DELTA,Q
   10 FORMAT(4I4,2F10.3)
        WRITE(6,137)IMAX,JMAX,N,NE,DELTA,Q
  137 FORMAT('1',////,4I5,F10.4,F10.4)
        READ(5,11)ARESM,MITR,EPSLN,ALERO
   11 FORMAT (F10.3,I4,2F10.3)
        WRITE(6,138)ARESM,MITR,EPSLN,ALERO
  138 FORMAT('0',F10.4,I5,2F10.4)
        READ (5,12)(X(I),I=1,N)
   12 FORMAT (8F10.4)
        WRITE(6,145)(X(I),I = 1,N)
  145 FORMAT('0',4E13.3)
        READ (5,12) (Y(I),I=1,N)
        WRITE(6,145)(Y(I),I = 1,N)

C       INITIALIZATION OF FREE SURFACE AND INTERNAL GRID
C       POTENTIALS

        J = 1
        DO 20 I = 1,IMAX
        H(I,J) = 1400.
   20 CONTINUE
        DO 31 I = 1,N
        HS(I,1) = Y(I)
   31 CONTINUE
        DO 25 I =1,IMAX
        DO 25 J = 2,JMAX
        H(I,J) = 500.
   25 CONTINUE
         NM = N-28
        DO 26 I =NM,N
        HS(I,1) = 1000.
   26 CONTINUE
        H(IMAX,JMAX) = 1000.
        WRITE (6,30)(HS(I,1),I=1,N)
   30 FORMAT ('0',4E13.3)
        JCHK = 0
  777 JCHK = JCHK + 1
        IF(JCHK-160)991,992,992
```

MAIN ...(CONT'D)

```
991 DO 755 JMP = 1,N
    CHK(JMP,JCHK)=HS(JMP,1)
755 CONTINUE
    IF(JCHK-1)2,754,2
  2 ERMAX = 0.0
    DO 733 JERR = 12,44
    ERR1 = ABS(CHK(JERR,JCHK)-CHK(JERR,JCHK-1))
    IF(ERR1 - ERMAX)733,711,711
711 ERMAX = ERR1
733 CONTINUE
    IF(ERMAX-ALERO)700,700,754
700 ICHK=0
    GO TO 789
754 CALL RELAX(IMAX,JMAX,EPSLN,ARESM,MITR)
    CALL PINT(IMAX,JMAX)
    CALL SOLVE(NE,IMAX,JMAX)
    GO TO 777
789 WRITE (6,100)
100 FORMAT('1',10X,'POTENTIALS AT GRID POINTS',///)
    DO 103 I=1,IMAX
    WRITE (6,101)(H(I,J),J=1,JMAX)
101 FORMAT (8(F10.4,5X))
103 CONTINUE
    WRITE (6,105)
105 FORMAT ('0',20X,'POTENTIALS AT THE FREE SURFACE',////)
    WRITE (6,106)(HS(I,1),I=1,N)
106 FORMAT (8(F10.4,5X))
    WRITE (6,107)
107 FORMAT('1',20X,'COORDINATES OF THE POINTS ON FREE
   * SURFACE',///)
    DO 108 I=1,N
    WRITE (6,109) Y(I),X(I)
109 FORMAT ('0',10X,F10.4,5X,F10.4)
108 CONTINUE
992 WRITE(6,993)
993 FORMAT('0',10X,'FAILED TO CONVERGE IN160 ITERATIONS')
    STOP
    END
```

SUBROUTINE RELAX

```
      SUBROUTINE RELAX(IMAX,JMAX,EPSLN,ARESM,MITR)
      COMMON H(30,60),MS(60,3),X(60),Y(60),RES(500),CHK(60
     *,160),KGP(25),
     1NPP(60)
      COMMON N,Q,DELTA
C     THIS SUBROUTINE CALCULATES POTENTIALS BY USING SOR
      LUT = 1
      DO 250 IP = 1,25
      KGP (IP) = 0
  250 CONTINUE
      NPP(1) = 0
      DO 13 LUP = 2,N
      XK = X(LUP)
    9 ALAX = XK/DELTA
      IF(ALAX - 1.)13,14,16
   16 XK = XK - DELTA
      GO TO 9
   14 YJ = Y(LUP)
   10 ALAY = YJ/DELTA
      IF(ALAY - 1.)13,39,40
   39 KGP(LUT) = LUP
      LUT = LUT + 1
      GO TO 13
   40 YJ = YJ - DELTA
      GO TO 10
   13 CONTINUE
      M = 2
      YY = Y(1)
      LUT = 1
      DO 50 L = 2,N
      IF ((YY-Y(L))-DELTA) 51,52,52
   51 NPP(M) =L
      M =M+1
      GO TO 50
   52 YY = YY-DELTA
      IF (L-KGP(LUT)) 50,53,50
   53 NPP(M) = L
      LUT = LUT+1
      M = M + 1
   50 CONTINUE
      KGPCH =1
      LCNT = 0
  222 HT = 0.0
      LCNT = LCNT+1
      YY = Y(1)
      J = 1
   27 J = J+1
      I = IMAX
```

SUBROUTINE RELAX ...(CONT'D)

```
   93 IF(J-(JMAX-1))54,261,260
  261 NM = N-1
      AN = (Y(NM)-Y(N))/DELTA
      HO = (HS(NM,1)/(1.+AN**2))+(AN**2/(2.*(1.+AN**2)))
     **(H(IMAX,JMAX-2)
     1+H(IMAX,JMAX))
      DH = ABS(HO-H(IMAX,JMAX-1))
      IF (HT-DH)88,87,87
   88 HT = DH
   87 H(IMAX,JMAX-1) = H(IMAX,JMAX-1)+EPSLN*(HO-H(IMAX,JMAX
     *-1))
      GO TO 27
   54 XX = DELTA
C     RELAX ALONG BASE LINE
      HO = (H(IMAX,J+1)+2.*H(IMAX-1,J)+H(IMAX,J-1))*.25
      DH = ABS(HO-H(IMAX,J))
      IF(HT-DH)231,232,232
  231 HT=DH
  232 H(IMAX,J)=H(IMAX,J)+EPSLN*(HO-H(IMAX,J))
   25 I = I - 1
   47 XX = XX+DELTA
      KJ = NPP(J)
      IF (XX-Y(KJ))20,21,21
C     RELAX INTERIOR POINTS
   20 HO=(H(I,J+1)+H(I-1,J)+H(I,J-1)+H(I+1,J))*0.25
      DH=ABS(HO-H(I,J))
      IF(HT-DH)215,214,214
  215 HT=DH
  214 H(I,J)=H(I,J)+EPSLN*(HO-H(I,J))
      GO TO 25
   21 KJ = NPP(J)
      IF ((X(KJ+1)-X(KJ))-DELTA)67,30,48
   67 IF (KJ-KGP(KGPCH))31,29,31
   31 EMDA = (X(KJ+1)-X(KJ))/DELTA
      AN = (DELTA-(XX-Y(KJ)))/DELTA
      HO=((2.*HS(KJ+1,1))/(EMDA*(1.+EMDA))+(2.*HS(KJ,1))/(AN
     **(1.+AN))+(2
     1.*H(I,J-1))/(1.+EMDA)+((2.*H(I+1,J))/(1.+AN)))*(EMDA
     **AN)/(2.*(EMDA
     1+AN))
      DH = ABS(HO-H(I,J))
      IF (HT-DH)18,19,19
   18 HT=DH
   19 H(I,J) = H(I,J)+EPSLN*(HO-H(I,J))
      GO TO 27
   29 EMDA = (X(KJ+1)-X(KJ))/DELTA
```

SUBROUTINE RELAX ...(CONT'D)

```
      HO=((2.*HS(KJ+1,1))/(EMDA*(1.+EMDA))+(2.*H(I,J-1))/(1.
     *+EMDA)+HS(KJ
     1,1)+H(I+1,J))*(EMDA/(2.*EMDA+2.))
      DH = ABS(HO-H(I,J))
      IF (HT-DH)41,42,42
   41 HT=DH
   42 H(I,J) = H(I,J)+EPSLN*(HO-H(I,J))
      KGPCH = KGPCH+1
      GO TO 27
   30 AN = (DELTA-(XX-Y(KJ)))/DELTA
      HO=(((2.*HS(KJ,1))/(AN*(1.+AN)))+((2.*H(I+1,J))/(1.
     *+AN))+H(I,J+1)+
     1H(I,J-1))*(AN/(2.*(AN+1.)))
      DH = ABS(HO-H(I,J))
      IF (HT-DH)99,22,22
   99 HT = DH
   22 H(I,J) = H(I,J)+EPSLN*(HO-H(I,J))
      GO TO 27
  260 IF (LCNT-MITR)270,270,273
  270 RES(LCNT)=HT
      IF (HT-ARESM)271,271,222
  271 WRITE(6,37) RES(LCNT),LCNT
   37 FORMAT ('0',20X,'MAX RESIDUE=',E12.5,2X,'ITERATION
     * NO=',I4)
      GO TO 273
   48 WRITE (6,60)
   60 FORMAT('1',10X,'UNEXPECTED HAPPENEDIN RELAX')
  273 RETURN
      END
```

SUBROUTINE PINT

```
      SUBROUTINE PINT(IMAX,JMAX)
C     THIS SUBROUTINE INTERPOLATES THE VALUES OF POTENTIALS
      DIMENSION KAN(20)
      COMMON H(30,60),HS(60,3),X(60),Y(60),RES(500),CHK(60
     *,160),KGP(25),
     1NPP(60)
      COMMON N,Q,DELTA
C     INITIALIZATION
      I=2
      NM3 = N-29
      KQQ = 1
      YY = Y(1)
      J = 1
      JJ=1
      KA=JJ
      JL = 1
      HS(1,2) = H(1,1)
      HS(1,3) = H(1,1)
      DO 30 LP = 1,20
      KAN(LP) = 0
   30 CONTINUE
   20 J=J+1
      JL = JL + 1
      KJ=J
      IF(J-(N-28)) 15 ,100,100
   15 IF((YY-Y(J)) - DELTA)1,1,3
    3 YY=YY-DELTA
      JL = JL - 1
      IF(J - KGP(KQQ))31,32,31
   32 HS(J,2) = H(I+1,JL)
      HS(J,3) = H(I+2,JL)
      KQQ = KQQ + 1
      GO TO 20
   31 WRITE(6,51)X(J),X(J-2),DELTA
   51 FORMAT('0',4X,'IN PINT',3F13.5)
      IF((X(J)-X(J-2))-DELTA)15,5,50
    1 IF((X(J)-X(J-1))-DELTA)4,5,50
C     INTERPOLATION IN Y-DIRECTIONS
    5 IF(KAN(KA)-J)25,17,25
   17 KJ = KJ + 1
   25 DO 6 I1=1,2
      NL = KJ + I1
      ML = I1 + 1
      HS(J, ML ) = HS(J,1) - ((HS(J,1)-H(I,JL))*(Y(J)-Y( NL
```

SUBROUTINE PINT ...(CONT'D)

```
     * ))/(DELTA-
     1(YY-Y(J))))
   36 IF((YY-Y(KJ+2))-DELTA)6,7,23
    7 KAN(JJ) = J
      JJ=JJ+1
      KA=JJ
      KAN(JJ) = J+1
      JJ=JJ+1
      KJ=J+1
   23 ML = ML+1
      IK = 2
   27 IF(J-NPP(IK))18,19,18
   18 IK = IK+1
      GO TO 27
   19 IF ((YY-DELTA)-(Y(KJ+2)+DELTA))28,152,28
   28 XY = ((YY-DELTA)-Y(KJ+2))
      GO TO 29
   29 HS(J,ML) = H(I,IK)-((H(I,IK)-H(I+1,IK))*XY)/DELTA
      GO TO 20
    6 CONTINUE
      GO TO 20
  152 HS(J,ML) = H(I+1,IK)
      GO TO 20

C     INTERPOLATION IN X-DIRECTION

    4 DO 10 I2=1,2
      NL1 = I2 + I
      ML1 = I2 + 1
      HS(J, ML1) = H( NL1,JL-1) - ((H(NL1 ,JL-1)-H( NL1,JL))
     **(DELTA-(X(K
     1J+1) - X(KJ)))/(X(KJ+1) - X(KJ-1)))
   10 CONTINUE
      I = I+1
      GO TO 20
   50 WRITE (6,60)
   60 FORMAT('1',10X,'DIFFERENCE IN X EXCEEDS DELTA IN PINT
     * SUBROUTINE')
  100 RETURN
      END
```

SUBROUTINE SOLVE

```
      SUBROUTINE SOLVE(NE,IMAX,JMAX)
      DIMENSION A(5,5),SA(25),C(5)
      COMMON H(30,60),HS(60,3),X(60),Y(60),RES(500),CHK(60
     *,160),KGP(25),
     1NPP(60)
      COMMON N,Q,DELTA
C     THIS SUBROUTINE SOLVES THE LINEAR EQUATIONS
      DO 10 I=1,NE
      DO 10 J=1,NE
      A(I,J)=0.0
   10 CONTINUE
      KQA = 1
      I=1
      KJ = 2
      JL = 2
      YY = Y(1)
      NM1 = N-28
      M=50
      A(1,1)=1.0
      A(1,4)=0.0
      A(1,5)=0.0
      A(2,1)=1.0
      A(2,4)=0.0
      A(2,5)=0.0
      A(3,1)=1.0
      A(3,2)=0.0
      A(3,3)=0.0
      A(4,1)=1.0
      A(4,2)=0.0
      A(4,3)=0.0
      A(5,1)=0.0
      A(5,3)=0.0
      A(5,5)=0.0
      DO 100 J = 2,NM1
      ALPHA= ATAN((Y(J-1)-Y(J))/(X(J)-X(J-1)))
      A(5,2)=SIN(ALPHA)
      A(5,4)=COS(ALPHA)
      IF(J-2)71,71,77
   71 DELT1 = DELTA
      DELT2 = DELTA
      DELT3 = DELTA- (YY - Y(J))
      DELT4 = DELTA
      C(1) = HS(1,1)
      C(2) = C(1)
      C(3) = H(2,2)
      C(4) = H(3,2)
      C(5) = 0.0
      GO TO 122
```

SUBROUTINE SOLVE ...(CONT'D)

```
 77 IF(X(J)-X(JL)-DELTA)11,12,13
 12 KJ=KJ+1
    KAJ=KJ
    JL=J
    DELT1 = DELTA
    DELT2 = DELTA
 11 IF(DELTA-(YY-Y(J)))14,15,16
 16 IF(M-1)20,21,22
 20 JAR1=J-1
    JAR2=JAR1
    M=M+1
    GO TO 40
 21 JAR1=J
    JAR2=J-1
    M=M+1
    GO TO 40
 22 JAR1=J
    JAR2=J
    M=M+1
 40 C(1)=HS(JAR1-1,2)
    C(2)=HS(JAR2-2,3)
    C(3)=H(I+1,KJ)
    C(4)=H(I+2,KJ)
    C(5) = 0.0
    DELT3 = DELTA - (YY - Y(J))
    DELT4 = DELTA
    GO TO 122
 15 M=0
    IF (J - KGP(KQA))27,26,27
 26 KAJ = KJ - 1
 27 C(1)=H(I+1,KAJ)
    C(2)=H(I+1,KAJ-1)
    C(3)=HS(J,2)
    C(4)=HS(J,3)
    C(5) = 0.0
    KAJ=KJ
    DELT3 = DELTA
    DELT4 = DELTA
    IF (J - KGP(KQA))29,28,29
 28 JA = J - 1
    KQA = KQA + 1
 29 JA = J
    DELT1 = X(J) - X(JA - 1)
    DELT2 = DELTA
    JA=J
    GO TO 122
 14 YY=YY-DELTA
    I=I+1
    GO TO 11
122 A(1,2) = -DELT1
```

SUBROUTINE SOLVE ...(CONT'D)

```
      A(1,3)=A(1,2)**2
      A(2,2) = -(DELT1+DELT2)
      A(2,3)=A(2,2)**2
      A(3,4) = -DELT3
      A(3,5)=A(3,4)**2
      A(4,4) = -(DELT3+DELT4)
      A(4,5)=A(4,4)**2
      CALL ARRAY(2,NE,NE,5,5,SA,A)
      CALL SIMQ(SA,C,NE,0)
      HS(J,1)=C(1)
  100 CONTINUE
      GO TO 18
   13 WRITE (6,114)
  114 FORMAT('0',10X,'UNEXPECTED HAPPENED IN SUBROUTINE
     * SOLVE')
   18 RETURN
      END
```

SUBROUTINE ARRA

```
      SUBROUTINE  ARRAY(MODE,NT,NB,NC,M,S,D)
      DIMENSION S(25),D(5,5)
      COMMON H(30,60),HS(60,3),X(60),Y(60),RES(500),CHK(60
     *,160),KGP(25),
     1NPP(60)
      COMMON N,Q,DELTA
  120 IP=0
      NM=0
      DO 130 K=1,NB
      DO 130 L = 1,5
      IP=IP+1
      S(IP) = D(L,K)
  130 CONTINUE
  140 RETURN
      END
```

SUBROUTINE SIMQ

```
      SUBROUTINE  SIMQ(A,B,NZ,KS)
      DIMENSION A(25),B(5)
      COMMON H(30,60),HS(60,3),X(60),Y(60),RES(500),CHK(60
     *,160),KGP(25),
     1NPP(60)
      COMMON N,Q,DELTA
      TOL = 0.0
      KS = 0
      JC=-NZ
      DO65JD=1,NZ
      JY=JD+1
      JC=JC+NZ+1
      BIGA = 0.
      IT = JC - JD
      DO30 ILT=JD,NZ
      IQ = IT + ILT
      IF(ABS(BIGA)-ABS(A(IQ)))20,30,30
   20 BIGA =A(IQ)
      IMAX = ILT
   30 CONTINUE
      IF(ABS(BIGA) - TOL)35,35,40
   35 KS = 1
      RETURN
   40 I1=JD+NZ*(JD-2)
      IT = IMAX -JD
      DO50 K=JD,NZ
      I1=I1+NZ
      I2 = I1 + IT
      SAVE = A(I1)
      A(I1) = A(I2)
      A(I2) = SAVE
   50 A(I1) = A(I1) / BIGA
      SAVE = B(IMAX)
      B(IMAX) =B(JD)
      B(JD) =SAVE/BIGA
      IF(JD-NZ)55,70,55
   55 IQS =NZ*(JD-1)
      DO65IX=JY,NZ
      IXJ = IQS + IX
      IT=JD-IX
      DO60 JX=JY,NZ
      IXJX =NZ*(JX-1)+IX
      JJX = IXJX + IT
   60 A(IXJX) = A(IXJX) - (A(IXJ)*A(JJX))
   65 B(IX)=B(IX)- (B(JD)*A(IXJ))
   70 NY=NZ-1
      IT=NZ*NZ
      DO80 JD=1,NY
      IA=IT-JD
      IB=NZ-JD
```

SUBROUTINE SIMQ ...(CONT'D)

```
      IC=NZ
      DO80 K= 1,JD
      B(IB) = B(IB) - A(IA)*B(IC)
      IA = IA-NZ
   80 IC = IC - 1
      RETURN
      END
```

APPENDIX B

SAMPLE CALCULATION FOR DISCHARGE

Sample Calculation for Discharge

Applying Darcy's law for the calculation of seepage discharge, per unit length in an isotropic material caused by a difference in head H, and using the notations shown in Fig. 2.1, it follows:

$$\Delta Q = k \frac{\Delta h}{\Delta s} \Delta n \tag{1}$$

$$q = N_F \Delta Q \quad \text{and} \quad H = N_d \Delta h \tag{2}$$

where N_F and N_d are the number of flow channels and head drops respectively.

From equations (1) and (2)

$$q = K.H. \frac{N_F}{N_a} \frac{\Delta n}{\Delta s}$$

For a squared flow-net $\frac{\Delta n}{\Delta s} = 1$. Therefore,

$$q = K.\Delta h \times N_F \tag{3}$$

where K = coefficient of permeability

Δh = head drop for each equipotential line.

The discharge calculations are given here for the case corresponding to Fig. 3.1 and q is denoted by q_{max}. Referring to Fig. 3.1, it follows

$$\begin{aligned} H &= 1000 \\ d &= 0 \\ \Delta h &= 50 \\ &= 0.05H \\ N_F &= 2.635. \end{aligned}$$

Therefore,

$$\begin{aligned} q_{max} &= k \times 0.05H \times N_F \\ &= k \times 0.05H \times 2.635 \end{aligned}$$

or,

$$\begin{aligned} \frac{q_{max}}{KH} &= 2.635 \times 0.05 \\ &= 0.1317. \end{aligned}$$

This is the dimensionless representation of the seepage discharge.